

This question paper contains 4+2 printed pages]

Your Roll No.....

3791

B.A. Prog./I

IS

(D)

OPERATIONAL RESEARCH

Paper I—Foundations of Operational Research

(Admissions of 2004 and onwards)

Time : 3 Hours

Maximum Marks : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

All Sections are compulsory and have equal marks.

Attempt any two parts from each Section.

Section I

1. (a) Explain the nature of operations/research and its limitations.
- (b) A ship is to carry 3 types of liquid cargo-X, Y and Z. There are 3,000 litres of X available, 2000 litres of Y available and 1500 litres of Z available. Each litre of X, Y and Z sold fetches a profit of

P.T.O.

Rs. 20, Rs. 35 and Rs. 40 respectively. The ship has 3 cargo holds A, B and C of capacities 2,000, 2,500 and 3,000 litres respectively. From stability consideration, it is required that each hold be filled in the same proportion. Formulate the problem.

- (c) Define and explain the following terms in an O.K. model :
- (i) Objective function
 - (ii) Decision variables.

Section II

2. (a) Solve the following system of equations :

$$x_1 + x_2 + x_3 = 2$$

$$x_1 + 2x_2 + 3x_3 = 3$$

$$2x_1 + 3x_2 + 4x_3 = 5.$$

- (b) Express the following matrix as a product of elementary matrices :

$$\begin{bmatrix} 2 & 0 & 3 \\ 0 & 1 & 2 \\ 1 & 0 & 3 \end{bmatrix}$$

- (c) Show that if λ is an eigen value of A , then $\alpha\lambda$ is that of αA , where α is a scalar with the same eigen vector.

Section III

3. (a) Examine whether the set

$$S = \{(x_1, x_2) : x_1^2 + x_2^2 < 1, x_1 + x_2 > 1\}$$

is convex set or not.

- (b) Define :

(i) Degenerate solution

(ii) Open and closed half space

(iii) Extreme point.

- (c) Define a basic solution to a given system of m simultaneous linear equations in n unknowns. Obtain all the basic solutions to the following system of linear equations :

$$x_1 + 2x_2 + x_3 = 4$$

$$2x_1 + x_2 + 5x_3 = 5.$$

Section IV

4. (a) Find the median of the following distribution.

Wages (in Rs.)	No. of Workers
2000-3000	3
3000-4000	5
4000-5000	20
5000-6000	10
6000-7000	5

- (b) The first three moments of a distribution about the value 2 are 1, 16 and - 40 respectively. Examine the skewness of the distribution.
- (c) State and prove Bayes' theorem.

Section V

5. (a) Let X be a continuous random variable with p.d.f. given by :

$$f(x) = \begin{cases} kx & 0 \leq x < 1 \\ k & 1 \leq x < 2 \\ -kx + 3k & 2 \leq x < 3 \\ 0 & \text{elsewhere} \end{cases}$$

- (i) Determine the constant k
- (ii) $E[X]$.
- (b) Prove that :
- (i) $E(X) < E(Y)$, where $P(X \leq Y) = 1$
- (ii) $|E(X)| \leq E(|X|)$.
- (c) Explain, under what conditions and how the binomial distribution can be approximated to the normal distribution.

Section VI

6. (a) If X and Y are independent normal variables zero means and standard deviations 9 and 12, respectively and if $X + 2Y$ and $kX - Y$ are uncorrelated, find k .

(b) X and Y are discrete random variables. If $\text{Var}(X) = \text{Var}(Y) = \sigma^2$, $\text{Cov}(X, Y) = \sigma^2/2$, find :

(i) $\text{Var}(2X - 3Y)$

(ii) $\text{Corr}(2X + 3, 2Y - 3)$.

(c) According to the norms established for a reading comprehension test, eighth graders should average 84.3 with a standard deviation of 8.6. If 45 randomly selected eighth graders from a certain school district averaged 87.8, test the null hypothesis $\mu = 84.3$ for that school district against the alternative hypothesis $\mu > 84.3$ using $\alpha = 0.01$.