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974

Your Roll No.

B.A. Prog./I

A

OPERATIONAL RESEARCH

(T)

Paper – Foundation of Operational Research

(Admissions of 2004 and onwards)

Time : 3 Hours

Maximum Marks : 75

*(Write your Roll No. on the top immediately
on receipt of this question paper.)*

*All Sections are compulsory and
have equal marks. Attempt any two
parts from each Section.*

SECTION – I

1. (a) Define O.R. and discuss its characteristics.
- (b) A company has three operational departments (weaving, processing and packing) with capacity to produce three different types of clothes namely suitings, shirtings and woollens yielding a profit of Rs. 2/-, Rs. 4/- and Rs. 3/- per meter, respectively.

P.T.O.

One meter of suiting requires 3 minutes in weaving, 2 minutes in processing and 1 minute in packing. Similarly one meter of shirting requires 4 minutes in weaving, 1 minute in processing and 3 minutes in packing. One meter of woollen requires 3 minutes in each department. In a week, total run time of each department is 60, 40 and 80 hours for weaving, processing and packing, respectively. Formulate the linear programming problem to find the product mix to maximize the profit.

- (c) Define and explain the following terms in an O.R. model –
- (i) Objective function
 - (ii) Decision variables

SECTION – II

2. (a) Prove that

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ bc & ca & ab \end{vmatrix} = (a-b)(b-c)(c-a)$$

- (b) Prove that the following set of vectors form a basis $[1, -1, 0]$, $[0, 1, -1]$, $[0, 0, 1]$ in R^3 .

- (c) Determine the eigen values and eigen vector of

$$A = \begin{bmatrix} 0 & -1 \\ 1 & 2 \end{bmatrix}$$

SECTION - III

3. (a) Define
- (i) Convex Set
 - (ii) Linear Independence of vectors
 - (iii) Closed and open half spaces
- (b) Show that the intersection of a two convex sets is also a convex set.
- (c) Obtain all basic solutions to the following system of linear equations.

$$2x_1 + x_2 - x_3 = 2$$

$$3x_1 + 2x_2 + x_3 = 3$$

SECTION - IV

4. (a) Find the mean and variance of first n-natural numbers.
- (b) The mean of 5 observation is 4.4 and variance is 8.24. If three of the five observations are 1, 2 and 6 find the other two.

- (c) An urn contains 6 white, 4 red, 9 black balls. If 3 balls are drawn at random, find the probability that
- two of the balls drawn are white.
 - one ball of each colour is drawn.

SECTION - V

5. (a) A random variable X has the following probability function :

Values of X, x : 0 1 2 3 4 5 6 7

$p(x)$: 0 k $2k$ $2k$ $3k$ k^2 $2k^2$ $7k^2+k$

- Find k .
 - Evaluate $P(X \leq 6)$, $P(X \geq 6)$.
- (b) If X is a random variable and 'a' is a constant, then prove that
- $E[a \psi(X)] = a E[\psi(X)]$,
 - $E[\psi(X) + a] = E[\psi(X)] + a$,
- where $\psi(X)$ is a function of X .
- (c) Define Poisson distribution. Show that the mean and variance of the Poisson distribution are equal.

SECTION - VI

6. (a) State the mathematical form and properties of normal distribution. Show that the mean, the median and the mode coincide.

- (b) Calculate the Karl Pearson's coefficient of correlation between X and Y for the following data

| | | | | | | | |
|-----|---|---|---|----|----|----|----|
| X : | 1 | 3 | 4 | 5 | 7 | 8 | 10 |
| Y : | 2 | 6 | 8 | 10 | 14 | 16 | 20 |

- (c) Suppose that 100 tires of a certain brand lasted on the average 21,431 miles with the standard deviation of 1295 miles. Using $\alpha = 0.05$, test the null hypothesis $\mu = 22,000$ miles against the alternative hypothesis $\mu < 22000$.