This question paper contains 4 printed pages.]

Your Roll No. .....

881

B.A. Prog./I **A**

(L)

MATHEMATICS : Paper I

(Algebra and Calculus)

(Admissions of 2004 and onwards)

Time : 3 Hours **Maximum Marks : 75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Note : The maximum marks printed on the question paper are applicable for the students of the regular colleges (Cat. 'A'). These marks will, however, be scaled up proportionately in respect of the students of NCWEB at the time of posting of awards for compilation of result.

All questions are compulsory and carry equal marks.

Attempt any **two** parts from each question.

1. (a) If x, y, z are linearly independent elements of a vector space over $\mathbb{R}$, the field of real numbers, show that $x + 3y - 2z$, $2x + y - z$ and $3x + y + z$ are also linearly independent over $\mathbb{R}$.

(b) Reduce the following matrix to normal form and hence determine its rank

$$A = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 3 & 4 & 5 & 2 \\ 2 & 3 & 4 & 0 \end{pmatrix}$$

(c) Find the characteristic equation of the matrix

$$\begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \\ 1 & 2 & 0 \end{pmatrix},$$

and hence compute its inverse.

2. (a) Prove that
$64 \sin^3\theta \cos^4\theta = -\sin 7\theta - \sin 5\theta + 3 \sin 3\theta + 3 \sin \theta$.

(b) By expanding $(1 + z)^n$ by Binomial theorem and then putting $z = \cos 2\theta + i \sin 2\theta$, prove that

(i) $2^n \cos^n\theta \cos n\theta = \sum_{r=0}^{n} {}^nC_r \cos 2r\theta$

(ii) $2^n \cos^n\theta \sin n\theta = \sum_{r=0}^{n} {}^nC_r \sin 2r\theta$.

(c) Solve the equation $3x^3 + 11x^2 + 12x + 4 = 0$, given that the roots are in H.P. (Harmonic Progression).

3. (a) Examine for continuity at $x = a$ of the function f, where

$$f(x) = \begin{cases} \frac{x^2}{a} - a & , 0 < x < a \\ 0 & , \quad x = a \\ a - \frac{a^3}{x^2} & , \quad a < x \end{cases}$$

Also examine if the function is derivable at $x = a$.

(b) If $z = \log\left(\frac{x^4 + y^4}{x + y}\right)$, prove that

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = 3 .$$

(c) Find the n^{th} derivative of $y = \frac{x}{x^2 + a^2}$.

4. (a) Find the angle of intersection of cardioides

$$r = a(1 + \cos\theta), \quad r = b(1 - \cos\theta).$$

(b) Find the asymptotes of the curve

$$x^2y + xy^2 + xy + y^2 + 3x = 0$$

(c) Trace the curve

$$y^2(2a - x) = x^3.$$

5. (a) State Rolle's Theorem and prove that between any two consecutive zeros of $f(x) = \sin x$ there is exactly one zero of $g(x) = \cos x$.

(b) Show that total surface area of a right circular cylinder with a given volume is minimum when its height is equal to its diameter.

(c) Evaluate (i) $\lim_{x \to 0^+} (\sin x)^x$

(ii) $\lim_{x \to 0} \dfrac{1 - \dfrac{x^2}{2} - \cos x}{x^2 \sin^2 x}$

6. (a) Obtain a reduction formula for

$$\int_0^{\infty} e^{-ax} x^n \, dx \quad (a > 0, n \in \mathbb{N})$$

(b) Find the arc length of the portion of the parabola $y^2 = 4ax$ cut off by the ordinate $x = a$.

(c) Find the volume of the solid generated by rotating the ellipse $4x^2 + y^2 = 4$ about the x-axis.

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