This question paper contains 4 printed pages.]

Your Roll No. ....

882

B.A. Prog./I A

(T)

MATHEMATICS : Paper I

(Algebra and Calculus)

(Admissions of 2004 and onwards)

Time : 3 Hours **Maximum Marks : 75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Note : The maximum marks printed on the question paper are applicable for the students of the regular colleges (Cat. 'A'). These marks will, however, be scaled up proportionately in respect of the students of NCWEB at the time of posting of awards for compilation of result.

All questions are compulsory and carry equal marks.

Attempt any **two** parts from each question.

1. (a) Define linear independence of a set of vectors in a vector space over a field. Show that the vectors (1, 2, 0), (0, 3, 1) and (–1, 0, 1) in $\mathbb{R}^{(3)}$ are linearly independent over $\mathbb{R}$.

(b) Find the rank of the matrix

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 3 & -2 & 1 \\ 2 & 0 & -3 & 2 \\ 3 & 3 & 0 & 3 \end{pmatrix}.$$

(c) Verify Cayley Hamilton Theorem for the matrix $A = \begin{pmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{pmatrix}$ and use it to find the inverse of A i.e. A^{-1}.

2. (a) Form an equation of the lowest degree with real coefficients, which has 1 + 2i and 3 – i as two of its roots.

(b) If α, β, γ are the roots of the equation

$x^3 + px^2 + qx + r = 0$, find the values of

(i) $\Sigma\alpha^2$

(ii) $\Sigma\alpha^2\beta^2$

(iii) $\Sigma\alpha^2\beta$

(c) Solve the equation

$2x^3 - 7x^2 + 7x - 2 = 0$, given that the roots are in G.P. (Geometric Progression).

3. (a) Determine whether $f(x)$ is continuous and has a derivative at the origin where

$$f(x) = 2 + x \quad \text{if } x \geq 0$$
$$= 2 - x \quad \text{if } x < 0$$

(b) If $V = r^m$, where $r^2 = x^2 + y^2 + z^2$,

show that

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = m(m+1)r^{m-2}.$$

(c) If $x = \sin t$, $y = \sin pt$, prove that

$$(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + p^2y = 0.$$

4. (a) Find the pedal equation of the parabola

$$y^2 = 4a(x + a).$$

(b) Find the asymptotes of the curve

$$x^3 + 3x^2y - 4y^3 - x + y + 3 = 0$$

(c) Trace the curve

$$y^2x = a^2(a - x).$$

5. (a) State and prove Lagranges' Mean Value theorem.

(b) Show that the semivertical angle of the right circular cone of given total surface and maximum volume is $\sin^{-1}\frac{1}{3}$.

(c) Evaluate (i) $\lim\limits_{x \to 0^+} (\sin x)^x$

(ii) $\lim\limits_{x \to 0} \frac{1 - \cos x}{x \sin x}$

6. (a) Evaluate $\int \frac{x^2 + 4x}{\sqrt{x^2 + x + 2}}\, dx$.

(b) Find the area of the region bounded by the asteroid $x^{2/3} + y^{2/3} = a^{2/3}$.

(c) Find the volume of the solid generated by rotating the ellipse

$$9x^2 + 4y^2 = 36 \text{ about } x\text{-axis.}$$
