[This question paper contains 4 printed pages.]

5546 *Your Roll No.* ....

B.A. Prog. / I Sem. **B**

(L)

MATHEMATICS : Paper A

Calculus

(Admissions of 2011 and onwards)

Time : 3 Hours *Maximum Marks : 75*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any **two** *parts from each question.*

SECTION I

1. (a) (i) Find a value for the constant k, if possible, that makes the function f continuous everywhere, where f is defined by

$$f(x) = \begin{cases} 7x - 2, & x \le 1 \\ kx^2, & x > 1 \end{cases} \qquad (6)$$

(ii) Show that $\lim_{x \to 0} x \sin\frac{1}{x} = 0$.

(b) Show that the function f defined by

$$f(x) = \begin{cases} x\left(\dfrac{e^{\frac{1}{x}} - e^{-\frac{1}{x}}}{e^{\frac{1}{x}} + e^{-\frac{1}{x}}}\right), & x \neq 0 \\ 0, & x = 0 \end{cases}$$

is not derivable at $x = 0$. (6)

(c) Prove that every differentiable function is continuous. Is the converse true? Justify your answer. (6)

2. (a) If $y = \dfrac{x^2}{(x+2)(2x+3)}$, find y_n. (6½)

(b) If $\cos^{-1}\left(\dfrac{y}{b}\right) = \log\left(\dfrac{x}{n}\right)^n$, prove that

$$x^2 y_{n+2} + (2n+1)x\, y_{n+1} + 2x^2 y_n = 0$$ (6½)

(c) If $U = x^2 \tan^{-1}\dfrac{y}{x} - y^2 \tan^{-1}\dfrac{x}{y}$, $x \neq 0, y \neq 0$, prove that

$$\frac{\partial^2 U}{\partial x \partial y} = \frac{x^2 - y^2}{x^2 + y^2}$$ (6½)

SECTION II

3. (a) If $p = x\cos\alpha + y\sin\alpha$ touches the curve

$\dfrac{x^m}{a^m} + \dfrac{y^m}{b^m} = 1$, prove that

$$p^{\frac{m}{m-1}} = (a\cos\alpha)^{\frac{m}{m-1}} + (b\sin\alpha)^{\frac{m}{m-1}}.$$ (6)

(b) If p_1, p_2 be the radii of curvature at the extremities of any chord of the cardioide $r = a(1 + \cos\theta)$ which passes through the pole, then prove that

$$\rho_1^2 + \rho_2^2 = \frac{16a^2}{9}. \quad (6)$$

(c) Find the angle of intersection of the parabolas $y^2 = 4ax$ and $x^2 = 4by$, at their point of intersection other than the origin. (6)

4. (a) Find the asymptotes of the curve

$$x^3 + 2x^2y + xy^2 - 2y^3 + xy - y^2 - 1 = 0 \quad (6½)$$

(b) Find the position and nature of the double points on the curve $y^2 = (x-a)^2(x-b)$. (6½)

(c) Trace the curve $y^2(x^2 - 1) = 2x - 1$. (6½)

SECTION III

5. (a) State and prove Lagrange's Mean Value Theorem and give its geometrical interpretation. (6½)

(b) Find Maclaurin's power series expansion of the function $f(x) = \sin x$. (6½)

(c) Show that

$$1 - \frac{x^2}{2} \leq \cos x \leq 1 - \frac{x^2}{2} + \frac{x^4}{24} \quad \forall x \in R \quad (6½)$$

6. (a) Determine the values of a and b for which

$$\lim_{x \to 0} \frac{x(1 + a\cos x) - b\sin x}{x^3}$$

exists and equals 1. (6)

(b) Show that x^x is a minimum for $x = e^{-1}$. (6)

(c) Separate the intervals in which the polynomial

$$2x^3 - 15x^2 + 36x + 1$$

is increasing or decreasing. (6)