This question paper contains 4 printed pages]

Your Roll No. ....

5547

B.A. Prog./I Sem. B

(T)

MATHEMATICS : Paper A

(Calculus)

(Admissions of 2004 and onwards)

Time : 3 *Hours* *Maximum Marks* : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any *two* parts from each question.

1. (*a*) Prove that :

$$\lim_{x \to 0} \frac{x - |x|}{x}$$

does not exist. 6

(*b*) A function f is defined by :

$$f(x) = \begin{cases} -x^2 & \text{if} \quad x \le 0 \\ 5x - 4 & \text{if} \quad 0 < x \le 1 \\ 4x^2 - 3x & \text{if} \quad x > 1 \end{cases}$$

Examine f for continuity at $x = 0$ and 1. Also discuss the kind of discontinuity, if any. 6

(*c*) Show that the function f defined by :

$$f(x) = |x| + |x - 1| + |x - 2|$$

is not derivable at $x = 0$, 1 and 2 but is continuous for every real number. 6

2. (*a*) Find the nth derivate of $e^x \sin^4 x$. 6½

(*b*) If $y = \cos(m \sin^{-1} x)$, prove that :

$$\left(1 - x^2\right) y_{n+2} - (2n + 1)\, xy_{n+1} + \left(m^2 - n^2\right) y_n = 0.$$

6½

(*c*) If

$$U = \frac{1}{\sqrt{x^2 + y^2 + z^2}},\ x^2 + y^2 + z^2 \neq 0,$$

show that :

$$\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial z^2} = 0.$$

6½

3. (*a*) Show that the length of the portion of the tangent to the curve $x = a \cos^3 \theta$, $y = a \sin^3 \theta$ intercepted between the co-ordinate axes is a constant. 6

(b) Find the condition for the curves $ax^2 + by^2 = 1$ and $a'x^2 + b'y^2 = 1$ to intersect orthogonally. 6

(c) The tangents at two points P, Q on the cycloid $x = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$ are at right angles, show that if P_1, P_2 be the radii of curvature at these points, then $P_1^2 + P_2^2 = 16a^2$. 6

4. (a) Find the asymptotes of the curve :

$$4x^3 - 3xy^2 - y^3 + 2x^2 - xy - y^2 - 1 = 0.$$ 6½

(b) Find the position and nature of the double points on the curve $y^2 = (x - 1)(x - 2)^2$. 6½

(c) Trace the curve $y^2\left(a^2 - x^2\right) = a^3 x$. 6½

5. (a) Show that :

$$\frac{v - u}{1 + v^2} < \tan^{-1} v - \tan^{-1} u < \frac{v - u}{1 + u^2},$$

$0 < u < v$ and deduce that :

$$\frac{\pi}{4} + \frac{3}{25} < \tan^{-1}\frac{4}{3} < \frac{\pi}{4} + \frac{1}{6}.$$ 6

(*b*) Find Maclaurin's power series expansion of the function :

$$f(x) = \log(1 + x) \text{ for } -1 < x \leq 1. \quad 6$$

(*c*) Prove that between any two real roots of $e^x \sin x = 1$, there is at least one real root of $e^x \cos x + 1 = 0$. 6

6. (*a*) State and prove Cauchy's mean value theorem. 6½

(*b*) If the limit of

$$\frac{\sin 3x - a \sin x}{x^3}$$

as $x \to 0$ is finite, find the value of a and the limit. 6½

(*c*) Show that the maximum value of $\left(\frac{1}{x}\right)^x$ is $e^{\frac{1}{e}}$. 6½