[This question paper contains 4 printed pages.]

5269 *Your Roll No.* ....

B.A. Prog. / I **B**

(L)

MATHEMATICS : Paper I

(Algebra and Calculus)

(Admissions of 2004 and onwards)

Time : 3 Hours *Maximum Marks : 75*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Note :– The maximum marks printed on the question paper are applicable for the students of the regular colleges (Cat. 'A'). These marks will, however, be scaled up proportionately in respect of the students of NCWEB at the time of posting of awards for compilation of result.

All *questions are compulsory and carry equal marks. Attempt any* **two** *parts from each question.*

SECTION I

1. (a) Define Subspace of a vector space over a field F. Prove that a nonempty subset W of a vector space V over F is a subspace of V if and only if

$\alpha w_1 + \beta w_2 \in W \ \forall \ \alpha, \beta \in F$ and $w_1, w_2 \in W$.

(b) Find the rank of the matrix

$$\begin{pmatrix} 2 & -3 & 1 & 1 \\ 1 & 2 & -3 & 0 \\ 4 & -1 & -2 & 2 \end{pmatrix}$$

by reducing it to its normal form.

(c) Verify that the matrix

$$A = \begin{pmatrix} 1 & 2 & 1 \\ 0 & 1 & -1 \\ 3 & -1 & 1 \end{pmatrix}$$

satisfies its characteristic equation. Hence find its cube.

SECTION II

2. (a) If $z = \cos\theta + i\sin\theta$, show that

$$\frac{z^{2n} - 1}{z^{2n} + 1} = i\tan n\theta,$$ n being an integer.

(b) Prove that

$$32 \sin^4\theta\cos^2\theta = \cos 6\theta - 2\cos 4\theta - \cos 2\theta + 2.$$

(c) If the sum of two roots of the equation

$$4x^4 - 24x^3 + 31x^2 + 6x - 8 = 0$$

be zero, find all the roots of the equation.

SECTION III

3. (a) Discuss the continuity of the function

$$f(x) = \begin{cases} 1 \ , & x \le 0 \\ 3-x, & 0 < x \le 1 \\ \dfrac{4}{x+1}, & 1 < x \end{cases}$$

at $x = 0$ and $x = 1$. Identify the type of discontinuity, if any.

(b) If $y = x^{n-1} \log x$, then show that $y_n = \dfrac{(n-1)!}{x}$.

(c) State Euler's Theorem for homogeneous functions and verify it for the function $z = \tan^{-1}(y/x)$.

SECTION IV

4. (a) Prove that the equation of the normal to the asteroid

$$x^{2/3} + y^{2/3} = a^{2/3}$$

may be written in the form

$$x \sin\phi - y \cos\phi + a \cos 2\phi = 0.$$

(b) For the curve $y = \dfrac{ax}{a + x}$, show that

$$\left(\frac{2\rho}{a}\right)^{2/3} = \left(\frac{y}{x}\right)^2 + \left(\frac{x}{y}\right)^2,$$

ρ, being the radius of curvature.

(c) Trace the curve $y^2(x^2 - 1) = 2x - 1$.

SECTION V

5. (a) State Lagrange's Mean Value theorem. Verify the Lagrange's mean value theorem for the function $f(x) = x(x-1)(x-2)$ in the interval $\left[0, \frac{1}{2}\right]$.

(b) Find the maxima and minima of the function

$$f(x) = \sin x + \frac{1}{2}\sin 2x + \frac{1}{3}\sin 3x$$

for all $x \in [0, \pi]$.

(c) Evaluate

$$\lim_{x\to 0} \frac{x^2 + 2\cos x - 2}{x \sin^3 x}.$$

SECTION VI

6. (a) Show that $\int_0^\pi \frac{dx}{(2+\cos x)^2} = \frac{2\pi}{3\sqrt{3}}$.

(b) Show that the area of the loop of the curve

$$y^2(a+x) = x^2(3a-x)$$

is equal to the area between the infinite branches of the curve and its asymptote.

(c) Find the volume of the solid generated by the loop of the curve $y^2(a+x) = x^2(a-x)$ about x-axis.