[This question paper contains 4 printed pages.]

5270 *Your Roll No.* .....

B.A. Prog. / I **B**

(L)

MATHEMATICS : Paper I

(Algebra and Calculus)

(Admissions of 2004 and onwards)

Time : 3 Hours *Maximum Marks : 75*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Note :– The maximum marks printed on the question paper are applicable for the students of the regular colleges (Cat. 'A'). These marks will, however, be scaled up proportionately in respect of the students of NCWEB at the time of posting of awards for compilation of result.

All *questions are compulsory and carry equal marks. Attempt any* **two** *parts from each question.*

1. (a) If x, y, z are linearly independent elements of a vector space over $\mathbb{R}$, the field of real numbers, show that $x + 3y - 2z$, $2x + y - z$ and $3x + y + z$ are also linearly independent.

P.T.O.

(b) Is the following system of equations

$$2x + 3y + 4z = 11$$

$$x + 5y + 7z = 15$$

$$3x + 11y + 13z = 25$$

consistent? If yes, then solve it.

(c) Verify Cayley Hamilton Theorem for the matrix

$$A = \begin{pmatrix} 0 & 0 & 1 \\ 3 & 1 & 0 \\ 2 & 1 & 4 \end{pmatrix}.$$

Hence compute A^{-1}, the inverse of A.

2. (a) If $\sin\theta + \sin\phi + \sin\psi = 0 = \cos\theta + \cos\phi + \cos\psi$, show that

$$\sin 3\theta + \sin 3\phi + \sin 3\psi = 3\sin(\theta + \phi + \psi)$$

and $\cos 3\theta + \cos 3\phi + \cos 3\psi = 3\cos(\theta + \phi + \psi)$.

(b) Prove that

$$32 \sin^4\theta\cos^2\theta = \cos 6\theta - 2\cos 4\theta - \cos 2\theta + 2.$$

(c) Solve the equation

$x^3 - 13x^2 + 15x + 189 = 0$, given that one of the roots exceeds another by 2.

3. (a) Show that the function f defined by $f(x) = |x| + |x-1|$ is continuous but not derivable at $x = 0$.

(b) If $z = \tan(y + ax) + (y-ax)^{3/2}$, find the value of $\frac{\partial^2 z}{\partial x^2} - a^2 \frac{\partial^2 z}{\partial y^2}$.

(c) If $y = \sin(\sin x)$, prove that

$$\frac{d^2y}{dx^2} + \tan x \frac{dy}{dx} + y\cos^2 x = 0.$$

4. (a) Find the pedal equation of the curve

$$r = a(1 + \cos\theta).$$

(b) Show that the asymptotes of the cubic

$$x^3 + 2x^2y - xy^2 - 2y^3 + 4y^2 + 2xy + y - 1 = 0$$

cut the curve in three points which lie on the straight line $x + 3y = 1$.

(c) Trace the curve

$$x(x^2 + y^2) = a(x^2 - y^2).$$

5. (a) State and prove Lagrange's Mean Value Theorem.

(b) Find the maximum value of perimeter of a right triangle of hypotenuse h.

(c) Evaluate (i) $\lim_{x \to 0^+} x^x$

(ii) $\lim_{x \to 0} \frac{1 - \cos x}{\sin^2 x}$

P.T.O.

6. (a) Evaluate $\int \frac{x^2 + 4x + 2}{\sqrt{x^2 + x + 1}} dx$.

(b) Find the area of region included between the parabolas $x^2 = 4ay$ and $y^2 = 4ax$.

(c) Find the volume of the solid of revolution obtained by revolving the hyperbola $xy = 1$, between $x = \frac{1}{n}$ and $x = n$ about X-axis.