[This question paper contains 4 printed pages.]

5602 *Your Roll No...........*

B.A. Prog. / II Sem. **B**

(L)

MATHEMATICS : Paper B

Algebra

(Admissions of 2011 and onwards)

Time : 3 Hours *Maximum Marks : 75*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any **two** *parts from each question.*

1. (a) Let V be the set of all real valued continuous functions defined on the closed interval [0, 1] and R be the field of real numbers, for any $f, g \in V$ and $\alpha \in R$ define $f+g$ and αf by

$$(f+g)(x) = f(x) + g(x)$$

$$(\alpha f)(x) = \alpha f(x), \; x \in [0, 1].$$

Prove that V is a vector space over R. (6)

(b) Define linearly dependent and linearly independent set of vectors. Prove that the set of vectors

(1, 2, 1), (1, 0, −1) and (0, −3, 2) are linearly independent. (6)

(c) Define subspace of a vector space over a field F. Prove that the intersection of two subspaces of a vector space V is a subspace of V. (6)

2. (a) Define rank of a matrix. Find rank of the following matrix

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 3 & 4 & 5 & 2 \\ 2 & 3 & 4 & 0 \end{pmatrix}$$ (6½)

(b) For what value of λ, the system of equations

$$\lambda x + 2y - 2z = 1$$
$$4x + 2\lambda y - z = 2$$
$$6x + 6y + \lambda z = 3$$

has a unique solution and then find the solution. (6½)

(c) Verify Cayley-Hamilton Theorem for the matrix

$$A = \begin{pmatrix} 1 & 2 & 1 \\ -1 & 0 & 3 \\ 2 & -1 & 1 \end{pmatrix}$$ (6½)

3. (a) Prove that

$$\cos 6\theta = \cos^6\theta - 15\cos^4\theta\sin^2\theta + 15\cos^2\theta\sin^4\theta - \sin^6\theta$$ (6)

(b) Solve the equation

$$x^6 + x^3 + 1 = 0$$

using De Moivre's theorem. (6)

(c) Sum upto n terms :–

$\cos\alpha + \cos(\alpha+\beta) + \cos(\alpha+2\beta) + \text{---} + n$ terms

provided $\beta \neq 2k\pi$. What is the value if $\beta = 2k\pi$? (6)

4. (a) Solve the equation

$$28x^3 - 39x^2 + 12x - 1 = 0$$

the roots being in H.P. (6½)

(b) The roots of the equation

$$16x^4 - 64x^3 + 56x^2 + 16x - 15 = 0$$

are in A.P. Find them. (6½)

(c) If α, β, γ be the roots of the equation

$x^3 + px + q = 0$, then show that

$$\frac{\alpha^5 + \beta^5 + \gamma^5}{5} = \frac{\alpha^3 + \beta^3 + \gamma^3}{3} \times \frac{\alpha^2 + \beta^2 + \gamma^2}{2}$$ (6½)

5. (a) Let n be a positive integer, then prove that a non zero element of Zn either has a multiplicative inverse or is a divisor of zero. Zn is the set of integers modulo n. (6)

(b) Consider the following permutations in S_7

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 3 & 2 & 5 & 4 & 6 & 1 & 7 \end{pmatrix} \text{ and}$$

$$\tau = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 2 & 1 & 5 & 7 & 4 & 6 & 3 \end{pmatrix}$$

Compute

(i) $\tau\sigma$ (ii) $\tau^{-1}\sigma\tau$ (6)

(c) Show that the set of all 2×2 matrices over R of the form $\begin{pmatrix} m & 0 \\ 0 & 1 \end{pmatrix}$ with $m \neq 0$, forms a group under matrix multiplication. (6)

6. (a) Let G be a non empty set with an associative binary operation in which the equations $ax = b$ and $xa = b$ have solutions for all $a, b \in G$. Then show that G is a group. (6½)

(b) Let G be a group and H be a non empty subset of G. Then H is a subgroup if and only if $ab^{-1} \in H$ for all $a, b \in H$. (6½)

(c) Prove that rigid motions of an equilateral triangle yield the group S_3. (6½)