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Your Roll No. ....

5603

B.A. Prog./II Sem. **B**

(T)

MATHEMATICS : Paper B

(Algebra)

(Admissions of 2011 and onwards)

Time : 3 Hours *Maximum Marks : 75*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any *two* parts from each question.

1. (*a*) Prove that the set of all matrices of the form $\begin{pmatrix} x & y \\ z & 0 \end{pmatrix}$, where $x, y, z \in C$ is a vector space over C with respect to matrix addition and multiplication of a matrix by a scalar, C denotes the set of complex numbers. 6

(*b*) Express vector $V = (3, 1, -4)$ as a linear combination of the vectors $v_1 = (1, 1, 1)$, $v_2 = (0, 1, 1)$, $v_3 = (0, 0, 1)$. Is the set $S = \{v, v_1, v_2, v_3\}$ linearly dependent ? 6

(*c*) Define bases of a vector space. If V is a vector space of dimension n, show that any set of n linearly independent vectors in V is a basis of V. 6

2. (*a*) Define rank of a matrix. Find rank of the following matrix :

$$\begin{pmatrix} 1 & 2 & -4 & 5 \\ 2 & -1 & 3 & 6 \\ 8 & 1 & 9 & 7 \end{pmatrix}$$ 6½

(*b*) For what values of λ and μ do the following system of equations :

$$x + y + z = 6$$

$$x + 2y + 3z = 10$$

$$x + 2y + \lambda z = \mu$$

have a unique solution. 6½

(*c*) Determine characteristic roots and characteristic vectors of the following matrix :

$$\begin{pmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{pmatrix}$$ 6½

3. (*a*) Prove that :

$$32\sin^4\theta\cos^2\theta = \cos 6\theta - 2\cos 4\theta - \cos 2\theta + 2$$ 6

(*b*) If α, β be the roots of $x^2 - 2x + 4 = 0$, prove that :

$$\alpha^n + \beta^n = 2^{n+1}\cos\frac{n\pi}{3}$$ 6

(*c*) Sum upto n terms :

$$\cos\alpha + \cos(\alpha - \beta) + \cos(\alpha - 2\beta) + \ldots\ldots\ldots n \text{ terms.}$$ 6

4. (*a*) Solve the equation :

$$x^3 - 9x^2 + 23x - 15 = 0$$

two of the roots being in the ratio 3 : 5. 6½ (*b*) Solve the equation :

$$x^4 + 2x^3 - 21x^2 - 22x + 40 = 0$$

the sum of two of the roots is equal to the sum of the other two. 6½

(*c*) If α, β, γ be the roots of the equation $x^3 + qx + r = 0$, find the values of : 6½

(*i*) $\sum \left(\frac{\beta}{\gamma} + \frac{\gamma}{\beta}\right)$

(*ii*) $\Sigma \alpha^2\beta$

5. (*a*) Let *a* and *b* be integers. Prove that in Z_n either $[a]_n \cap [b]_n = \phi$ *or* $[a]_n = [b]_n$. Z_n is the set of integers modulo *n*. 6

(*b*) Consider the following permutations in S_7 :

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 3 & 2 & 5 & 4 & 6 & 1 & 7 \end{pmatrix} \text{ and}$$

$$\tau = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 2 & 1 & 5 & 7 & 4 & 6 & 3 \end{pmatrix}$$

Compute the following :

(*i*) $\sigma\tau$

(*ii*) $\tau^2\sigma$ 6

(*c*) Let $S = \mathbf{R}\backslash\{-1\}$. Define * on S by :

$a * b = a + b + ab$

Show that (S, *) is a group. 6

6. (*a*) If G is a group and $a, b \in G$, then prove that the equation $ax = b$ and $xa = b$ have unique solution 6½.

(*b*) If H is a subgroup of a finite group G, then prove that order of H is a divisor of order of G. 6½

(*c*) Prove that the set {0, 1, 2, 3, 4, 5} with addition modulo 6 and multiplication modulo 6 as the composition is a commutative ring with unity. 6½