[This question paper contains 4 printed pages.]

**5595** *Your Roll No............*

**B.A. Prog./Sem. II** **B**

STATISTICS – Paper B

(Statistical Methods – I)

*Time : 3 Hours* *Maximum Marks : 75*

*(Write your Roll No. on the top immediately on receipt of this question paper.)*

*Question No.* **1** *is compulsory and carries* **15** *marks, other questions carry* **12** *marks each. Attempt* **six** *questions in all.*

1. (a) Fill in the blanks :

(i) For a symmetrical distribution $\beta_1$ = ______ .

(ii) For $N(\mu, \sigma^2)$, mean deviation about mean = ______ .

(iii) For Poisson distribution, all cumulants are ______ .

(iv) Mean = Variance, for ______ distribution (Discrete).

(v) If X and Y are independent random variables then Cov(X, Y) = ______ .

P.T.O.

(b) If $X \sim N(1, 4)$, $Y \sim N(2, 3)$ are independent random variables. What is the distribution of $X = 2Y$ ?

(c) If X and Y are independent random variables then $E[XY + Y + 1] - E(X+1)E(Y) =$ ______ .

(d) A random variable X has the probability distribution

| X : | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| p/x : | 0 | K | 2K | 2K | 3K | $K^2$ | $2K^2$ | $7K^2+K$ |

Find (i) K (ii) $P(X > 5)$.

(e) State the conditions under which

(i) Binomial distribution tends to normal distribution

(ii) Poisson distribution tends to normal distribution

(f) Identify the distributions with the following m.g.f.'s

(i) $M_X(t) = (0.3 + 0.7e^t)^{10}$ (ii) $e^{3(e^t - 1)}$

(5,2,2,2,2,2)

2. (a) Find the mean, variance and the coefficients $\beta_1$, $\beta_2$ of the distribution

$$dF(x) = Kx^2e^{-x}\,dx, \; 0 < x < \infty.$$

(b) Joint distribution of X and Y is given by

$$f(x, y) = 4xye^{-(x^2+y^2)}; \; x \geq 0, \; y \geq 0.$$

Test whether X and Y are independent ? Also find the conditional density of X given $Y = y$.

(6,6)

3. (a) State and prove addition law of expectation for two random variables X and Y.

(b) A coin is tossed until a head appears. What is the expectation of the number of tosses required ?

(6,6)

4. (a) State Chebychev's Inequality.

For geometric distribution $p(x) = 2^{-x}$, $x = 1, 2, 3, -$, prove that Chebychev's inequality gives

$$P[|X - 2| \leq 2] > \frac{1}{2}$$

while the actual probability is 15/16.

(b) State WLLN.

Examine whether the WLLN holds for the sequence $\{X_K\}$ of independent random variables defined as follows :

$$P\left(X_K = \pm 2^K\right) = 2^{-(2K+1)},\ P\left(X_K = 0\right) = 1 - 2^{-2K}$$ (6,6)

5. (a) State and prove De-Moivre's Laplace theorem.

(b) Let $X_1$, $X_2$, - - - be i.i.d. Poisson variates with parameter $\lambda$. Use CLT to estimate $P(120 \leq S_n \leq 160)$, where $S_n = X_1 + X_2 + - - + X_n$ $\lambda = 2$ and $n = 75$.

(6,6)

6. (a) Let $X \sim B(n, p)$. Show that

$$\mu_{r+1} = pq\left[nr\,\mu_{r-1} + \frac{d\mu_r}{dp}\right],$$

where $\mu_r = E\left[X - E(x)\right]^r$.

(b) If X and Y are independent Poisson variates such that

$$P(X=1) = P(X=2) \text{ and } P(Y=2) = P(Y=3).$$

Find the variance of $X - 2Y$. (6,6)

7. (a) Obtain the mean deviation about mean of normal distribution with mean $\mu$ and variance $\sigma^2$.

(b) Write down the Beta probability density function of the first kind with parameters $\mu$ and $\nu$. Hence obtain its Harmonic mean. (6,6)

8. (a) Obtain binomial distribution as a limiting form of hypergeometric distribution.

(b) If X has a uniform distribution in [0, 1], find the p.d.f. of $-2 \log X$. Identify the distribution.

(6,6)