[This question paper contains 4 printed pages.]

1181 *Your Roll No.* ....

B.A. Prog. / I **C**

MATHEMATICS : Paper I

(Algebra and Calculus)

(Admissions of 2004 and onwards)

Time : 3 Hours *Maximum Marks : 75*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Note :- The maximum marks printed on the question paper are applicable for the students of the regular colleges (Cat. 'A'). These marks will, however, be scaled up proportionately in respect of the students of NCWEB at the time of posting of awards for compilation of result.

All *questions are compulsory and carry equal marks. Attempt any* **two** *parts from each question.*

SECTION I

1. (a) Show that the set

$$Q(\sqrt{2}) = \{a + b\sqrt{2} : a, b \in Q\}$$

is a vector space over Q with respect to compositions :

$$(a + b\sqrt{2}) + (c + d\sqrt{2}) = (a + c) + (b + d)\sqrt{2}$$

$$\alpha(a + b\sqrt{2}) = \alpha a + \alpha b\sqrt{2}$$

for all a, b, c, d, $\alpha \in Q$.

P.T.O.

(b) Using elementary row transformations, find the

$$\text{rank of } A = \begin{pmatrix} 2 & 3 & 1 & -1 \\ 1 & -1 & -2 & 4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & 7 \end{pmatrix}.$$

(c) Which of the following matrices are skew-Hermitian ?

(i) $\begin{pmatrix} i & -1 & 2 \\ 1 & i & 3 \\ -2 & -3 & i \end{pmatrix}$

(ii) $\begin{pmatrix} 1 & 3 & 7+i \\ 3i & -i & 6 \\ 7-i & 8 & 0 \end{pmatrix}$

(iii) $\begin{pmatrix} 0 & 1-i & 2-3i \\ -1-i & 0 & 6i \\ -2-3i & 6i & 0 \end{pmatrix}$

SECTION II

2. (a) Sum the series :

$$\sin\alpha + c\sin(\alpha+\beta) + \frac{c^2}{2!}\sin(\alpha+2\beta) + \text{-----}$$

(b) If sum of the roots of the equation

$$4x^4 - 24x^3 + 31x^2 - 6x - 8 = 0$$

be zero, find all roots of the equation.

(c) If $\alpha + \beta + \gamma = 1$

$$\alpha^2 - \beta^2 + \gamma^2 = 2$$

$$\alpha^3 + \beta^3 + \gamma^3 = 3$$

find the value of $\alpha^4 + \beta^4 + \gamma^4$.

SECTION III

3. (a) Examine the continuity of the function

$$f(x) = \frac{x e^{1/x}}{1 + e^{1/x}}, \quad x \neq 0$$

$$= 0, \quad x = 0$$

at $x = 0$.

(b) If $y = (x^2 - 1)^n$, then prove that

$$(x^2 - 1)y_{n+2} + 2xy_{n+1} - n(n-1)y_n = 0.$$

(c) If $z = \tan^{-1}\left(\frac{x + y}{\sqrt{x} + \sqrt{y}}\right)$, then show that

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = \frac{1}{4}\sin 2z$$

SECTION IV

4. (a) Find the condition that the curves $ax^2 + by^2 = 1$ and $a^1x^2 + b^1y^2 = 1$ should cut orthogonally.

(b) Find the position and nature of the double points on the curve

$x^4 - 2ay^3 - 3a^2y^2 - 2a^2x^2 + a^4 = 0$

(c) Define a node and a cusp on a curve of the form $f(x, y) = 0$. Find the tangents at the origin to the curve $a^2(x^2 - y^2) = x^2y^2$.

SECTION V

5. (a) Obtain Mclaurin's Series expansion of $\log(1 + x)$.

(b) Show that the function (defined for $x > 0$) $f(x) = xe^{1/x}$ has only one extreme value. Find the corresponding point, value and its nature.

(c) Evaluate $\lim_{x \to 0} \frac{a - b\cos x - c\sin x}{x^2}$, given that the limit exists. Also find the values of a, b and c.

SECTION VI

6. (a) Evaluate $\int_0^{\pi/2} \log \sin x \, dx$.

(b) Find the arc length of the portion of the parabola $y^2 = 4ax$ cut off by its latus rectum.

(c) Find the volume of the solid obtained by revolving the ellipse

$$\frac{x^2}{4^2} + \frac{y^2}{2^2} = 1 \text{ about X-axis.}$$