[This question paper contains 4 printed pages.]

1180 *Your Roll No.* ....

B.A. Prog. / I **C**

MATHEMATICS : Paper I

(Algebra and Calculus)

(Admissions of 2004 and onwards)

Time : 3 Hours *Maximum Marks : 75*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Note :– The maximum marks printed on the question paper are applicable for the students of the regular colleges (Cat. 'A'). These marks will, however, be scaled up proportionately in respect of the students of NCWEB at the time of posting of awards for compilation of result.

All *questions are compulsory and carry equal marks. Attempt any* **two** *parts from each question.*

SECTION – I

1. (a) Show that the vectors (1,2,1), (2,1,0), (1,–1,2) form a basis of $\mathbb{R}^3$.

(b) Obtain the characteristic equation of the matrix $\begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{pmatrix}$ and find the inverse of the matrix.

(c) Solve the system of equations, if possible

$$x + y + z = 2$$
$$x - 2y + 3z = -5$$
$$x + 3y - 6z = 11$$
$$x + 4y + 10z = 21$$

SECTION – II

2. (a) If α, β are the roots of equation $x^2 - 2x + 4 = 0$, prove that

$$\alpha^n + \beta^n = 2^{n+1} \cos \frac{n\pi}{3}$$

and hence evaluate $\alpha^{n} + \beta^{n}$.

(b) Solve the equation $27x^3 + 42x^2 - 28x - 8 = 0$, the roots being in G.P.

(c) If α, β, γ are the roots of

$$x^3 + ax^2 + bx + c = 0,$$

evaluate (i) $\sum \left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right)$

(ii) $\sum \alpha^2$

SECTION – III

3. (a) Let f be the function defined on ℝ by setting

$$f(x) = \begin{cases} \dfrac{e^{1/x} - e^{-1/x}}{e^{1/x} + e^{-1/x}}, & x \neq 0 \\ 1, & x = 0 \end{cases}$$

show that f is continuous from the right at $x = 0$ and has a discontinuity of the first kind from the left.

(b) State and prove Leibnitz's Theorem.

(c) If $v = r^m$, where $r^2 = x^2 + y^2 + z^2$, show that

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = m(m+1)r^{m-2}$$

SECTION – IV

4. (a) Find the equation of the normal to the curve $3x^2 - y^2 = 8$ parallel to the line $x + 3y = 4$.

(b) Show that the radius of curvature of the curve $r^m = a^m \cos m\theta$ is :

$$\frac{a^m}{(m+1)r^{m-1}}$$

(c) Find the position and nature of the double points on the curve :

$$x^3 + 2x^2 + 2xy - y^2 + 5x - 2y = 0$$

SECTION – V

5. (a) Show that a circular cylinder with a given volume has a minimum total surface area if its height is equal to the diameter.

(b) State Lagrange's Mean Value Theorem, use it to prove that between any two consecutive zero's of $\cos x$ there is exactly one zero of $\sin x$.

(c) Evaluate

$$\lim_{x \to 0} \frac{e^x - e^{\sin x}}{x - \sin x}$$

SECTION – VI

6. (a) If $u_n = \int_0^{\pi/2} x^n \sin x \, dx$ $(n > 1)$, then prove that

$$u_n + n(n-1)u_{n-2} = n\left(\frac{\pi}{2}\right)^{n-1}$$

Deduce the value of u_5.

(b) Find the area of the region included by the parabolas $x = y^2$ and $y = x^2$.

(c) Find the volume of the solid obtained by revolving the loop of the curve

$a^2y^2 = x^2(x - a)(2a - x)$ about the X-axis.