[This question paper contains 4 printed pages.]

1271 *Your Roll No.* .....

B.A. Prog. / I **C**

(D)

OPERATIONAL RESEARCH

Paper I – Foundations of Operational Research

(Admissions of 2004 and onwards)

Time : 3 Hours *Maximum Marks : 75*

(Write your Roll No. on the top immediately on receipt of this question paper.)

All *Sections are compulsory and have equal marks. Attempt any* **two** *parts from each Section.*

SECTION I

1. (a) Describe briefly the origin and development of Operational Research.

   (b) A firm makes two products X and Y and has a total production capacity of 9 tons per day. The firm has a permanent contact to supply at least 2 tones of X and at least 3 tones of Y per day to another company. Each ton of X requires 20 machine hours production time and each ton of Y requires 50 machine hours of production time. The daily maximum possible number of machine hours is 360. All the firm's output can be sold, and the profit made is Rs. 80 per ton of X and Rs. 120

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per ton of Y. Formulate this problem as an LP model to determine the production schedule for maximum profit.

(c) Explain the concept, scope and tools of Operational research as applicable to business and industry.

SECTION II

2. (a) Find the inverse of the following 3X3 matrix using elementary row operations

$$A = \begin{pmatrix} 1 & 2 & -3 \\ 1 & -2 & 1 \\ 5 & -2 & -3 \end{pmatrix}$$

(b) Prove that any set of vectors containing the zero vector is linearly dependent.

(c) Solve, if consistent, the system of equations :

$$x + y + 3z = 1$$
$$2x + 3y - z = 3$$
$$5x + 7y + z = 7$$

SECTION III

3. (a) Are the vectors $X_1 = (1, 0, 1, 2)$, $X_2 = (0, 1, 1, 2)$ and $X_3 = (1, 1, 1, 3)$ in $\mathbf{R}^4$ linearly dependent or linearly independent ?

(b) Define a convex set and examine the convexity of the set :

$$s = \{(x, y) : y^2 \leq 4x\}$$

(c) What do you understand by basic feasible solution ? Determine all possible basic solutions for the following set of equations

$$x_1 + 2x_2 + x_3 = 4$$
$$2x_1 + x_2 + 5x_3 = 5$$

SECTION IV

4. (a) The first of the two samples has 100 items with mean 15 and standard deviation 3. If the whole group has 250 items with mean 15.6 and standard deviation $(13.44)^{1/2}$, find the standard deviation of the second group.

(b) An air-conditioning repair person claims that the probability is 0.82 that the motor is alright, 0.64 that the fan is alright and 0.41 that they both are alright. Do you think that he is justified in his claim ? Give reasons for your answer.

(c) State and prove theorem of total probability.

SECTION V

5. (a) Find the value of k so that the function f(x) defined as follows be a density function

$$f(x) = \begin{cases} k & \text{if } x = 0 \\ 2k & \text{if } x = 1 \\ 3k & \text{if } x = 3 \\ 0 & \textit{otherwise} \end{cases}$$

(b) In a distribution exactly normal, 7% of the items are under 35 and 89% are under 63. What are the mean and standard deviation of the distribution ?

(c) Two unbiased dice are thrown. Find the expected values of the sum of numbers of points on them.

SECTION VI

6. (a) A sample of 900 members has a mean 3.4 cms and a standard deviation 2.61 cms. Is the sample from a large population of mean 3.25 cms and standard deviation 2.61 cms. If the population is normal and its mean is unknown, find the 98% confidence limits of true mean.

(b) In a binomial model consisting of 5 independent trials, probabilities of 1 and 2 successes are 0.4096 and 0.2048 respectively. Find the parameter 'p' of the distribution.

(c) Find the most likely price in Mumbai corresponding to the price of Rs. 60 at Calcutta from the following data

	Calcutta	Mumbai
Average price	65	67
Standard deviation	2.5	3.5

Coefficient of correlation is 0.8 between the two prices of commodities in the two cities.

(100)