This question paper contains **4+1** printed pages]

Your Roll No.....

7560

B.A. Prog./I **D-I**

MATHEMATICS—Paper I

(Algebra and Calculus)

(NC : Admissions of 2006 onwards)

Time : 3 *Hours* *Maximum Marks* : 100

(Write your Roll No. on the top immediately on receipt of this question paper.)

All the *six* questions are compulsory and carry equal marks.

Attempt any *two* parts from each question.

1. (*a*) Define subspace of a vector space over a field F. Prove that a non-empty subset W of a vector space V over a field F is a subspace of V iff :

$\alpha w_1 + \beta w_2 \in W \quad \forall\ \alpha, \beta \in F$ and $w_1, w_2 \in W$

P.T.O.

(*b*) Find the rank of the matrix :

$$\begin{bmatrix} 1 & 3 & 6 & -1 \\ 1 & 4 & 5 & 1 \\ 1 & 5 & 4 & 3 \end{bmatrix}$$

(*c*) Find the characteristic equation of the matrix :

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 1 & 0 & -1 \end{bmatrix}$$

and hence compute its cube.

2. (*a*) Prove the following identity :

$$32\sin^4\theta\cos^2\theta = \cos 6\theta - 2\cos 4\theta - \cos 2\theta + 2.$$

(*b*) Solve the equation :

$$x^3 - 9x^2 + 23x - 15 = 0,$$

two of the roots being in the ratio 3 : 5

(*c*) If α, β, γ are the roots of the equation :

$$x^3 - px^2 + qx - r = 0,$$

Find the values of :

(*i*) $\Sigma\alpha^2\beta$

(*ii*) $\Sigma\alpha^3$

(*iii*) $\Sigma\frac{\alpha}{\beta}$

3 (*a*) Show that the function f where :

$$f(x) = \begin{cases} \dfrac{x^3 - 8}{x^2 - 4}, & x \neq 2 \\ 3, & x = 2 \end{cases}$$

is continuous at $x = 2$.

(*b*) If

$$z = \log\frac{x^4 + y^4}{x + y}.$$

then prove that :

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = 3$$

(*c*) If

$$y = \sin^{-1} x,$$

then show that :

$$(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - n^2 y_n = 0.$$

4. (*a*) Show that the Pedal equation of the parabola :

$$y^2 = 4a(x + a) \text{ is } p^2 = ar.$$

(*b*) Find the asymptotes of the curve :

$$x^3 + 2x^2y - xy^2 - 2y^3 + xy - y^2 - 1 = 0.$$

(*c*) Trace the curve :

$$ay^2 = x^2(a - x)$$

5. (*a*) Verify Lagrange's Mean Value theorem on the interval [1, 4] for the function :

$$f(x) = (x - 1)(x - 2)(x - 3).$$

(*b*) Find the minimum and maximum values of the function :

$$f(x) = x^5 - 5x^4 + 5x^3 - 1.$$

(*c*) Find :

$$\lim_{x \to 0} \frac{xe^x - \log(1 + x)}{x^2}$$

6. (*a*) Show that :

$$\int \sin^n x dx = -\frac{\cos x \sin^{n-1} x}{n} + \frac{n-1}{n} \int \sin^{n-2} x dx .$$

(*b*) Find the area bounded by the parabola $y^2 = 4ax$ and its latus rectum.

(*c*) Find the volume of the solid generated by the revolution of the curve $(a - x)y^2 = a^2x$ about its asymptote.