This question paper contains 4+2 printed pages]

Your Roll No. ....

7561

B.A. (Prog.)/I **D-I**

MATHEMATICS—Paper I

(Algebra and Calculus)

(Admissions of 2004 and onwards)

Time : 3 *Hours* *Maximum Marks* : 100

(Write your Roll No. on the top immediately on receipt of this question paper.)

Note :— The maximum marks printed on the question paper are applicable for the students of Category 'B'. These marks will, however, be scaled down proportionately in respect of the students of Regular Colleges, Category 'A' at the time of posting of awards for compilation of result.

All the *six* questions are compulsory and carry equal marks.

Attempt any *two* parts from each question.

P.T.O.

1. (*a*) Define linearly dependent and linearly independent set of vectors.

Prove that the set of vectors (1, 2, 0), (0, 3, 1) and (–1, 0, 1) in $R^{(3)}$ are linearly independent.

(*b*) Find the rank of the matrix :

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 3 & 4 & 5 & 2 \\ 2 & 3 & 4 & 0 \end{bmatrix}$$

(*c*) Find the characteristic equation of the matrix :

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$$

and hence calculate its inverse.

2. (*a*) If :

$$\cos\theta + 2\cos\varphi + 3\cos\psi = 0,$$

$$\sin\theta + 2\sin\varphi + 3\sin\psi = 0,$$

prove that :

$$\cos 3\theta + 8\cos 3\varphi + 27\cos 3\psi = 18\cos(\theta + \varphi + \psi) \text{ and}$$

$$\sin 3\theta + 8\sin 3\varphi + 27\sin 3\psi = 18\sin(\theta + \varphi + \psi).$$

(*b*) Solve the equation :

$$3x^3 - x^2 - 3x + 1 = 0$$

the roots being in H.P. (Harmonic Progression).

(*c*) If α, β, γ are the roots of the equation :

$$x^3 + px^2 + qx + r = 0,$$

find the values of :

(*i*) $\Sigma \frac{1}{\alpha}$

(*ii*) $\Sigma\alpha^2 \beta$

(*iii*) $\Sigma(\alpha - \beta)^2$

3. (*a*) Examine the continuity of the function defined by :

$$f(x) = |x - 1| \text{ at } x = 1.$$

(*b*) If

$$z = \tan^{-1}\frac{x^3 + y^3}{x - y},$$

then prove that :

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = \sin 2z.$$

(*c*) If

$$y = \sin(m \sin^{-1} x),$$

then prove that :

$$(1 - x^2)\, y_{n+2} = (2n + 1)xy_{n+1} + (n^2 - m^2)y_n.$$

4. (*a*) Prove that the sum of the intercepts on the coordinate axes of any tangent to the curve :

$$\sqrt{x} + \sqrt{y} = \sqrt{a}$$

is constant.

(*b*) Find the asymptotes of the curve :

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1.$$

(*c*) Trace the curve :

$$y^2 x = a^2(a - x).$$

5. (*a*) Verify Rolle's theorem on the interval [2, 4] for the function :

$$f(x) = (x - 2)(x - 3)(x - 4).$$

(*b*) Show that the maximum value of :

$$\left(\frac{1}{x}\right)^x \text{ is } e^{\left(\frac{1}{e}\right)}.$$

(*c*) Find :

$$\lim_{x \to 0} \frac{1 - \cos x^2}{x^2 \sin x^2}.$$

6. (*a*) Show that :

$$\int \sin^m x \cos^n x dx = \frac{\sin^{m+1} x \cos^{n-1} x}{m + n} + \frac{n - 1}{m + n} \int \sin^m x \cos^{n-2} x dx.$$

(*b*) Find the area between the curve :

$$y^2(2a - x) = x^3$$

and its asymptote.

(*c*) Find the volume formed by the revolution of the loop of the curve :

$$y^2(a + x) = x^2\ (a - x)$$

about x-axis.