

This question paper contains 4+2 printed pages]

Your Roll No.

7625

B.A. (Prog.)/I

D-II

MATHEMATICS—Paper I

(Algebra and Calculus)

(NC : Admissions of 2006 onwards)

Time : 3 Hours

Maximum Marks : 100

(Write your Roll No. on the top immediately on receipt of this question paper.)

Note :— The maximum marks printed on the question paper are applicable for the students of Category 'B'. These marks will, however, be scaled down proportionately in respect of the students of Regular Colleges, Category 'A' at the time of posting of awards for compilation of result.

Attempt any *two* parts from each question.

P.T.O.

Part A (Algebra)**Section I**

1. (a) Which of the following set of vectors is linearly dependent ? Justify. 9

(i) $S_1 = \{(1, 2, 0), (0, 3, 1), (-1, 0, 1)\}$ in \mathbf{R}^3 .

(ii) $S_2 = \{(1, 1, 0), (2, 1, 1), (3, 0, 3)\}$ in \mathbf{R}^3 .

- (b) Obtain the characteristic equation of the matrix :

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$$

and hence calculate its inverse. 9

- (c) Using matrices solve the following system of equations : 9

$$x + 2y - z = 3$$

$$3x - y + 2z = 1$$

$$2x - 2y + 3z = 2$$

$$x - y + z = -1$$

Section II

2. (a) Two roots of the equation :

$$x^3 - 4x^2 - 3x + 18 = 0$$

are equal, find its roots.

8

- (b) Prove the identity :

$$32\sin^6 \theta = -\cos^6 \theta + 6\cos^4 \theta - 15\cos^2 \theta + 10 \quad 8$$

- (c) If α, β, γ be the roots (all non-zero) of the equation :

$$x^3 - px^2 + qx - r = 0,$$

find the value of $\sum \frac{\alpha}{\beta}$.

8

Part B (Calculus)**Section III**

3. (a) Show that the function :

$$f(x) = x - |x|$$

is continuous at $x = 0$.

8

P.T.O.

(b) Find the n th derivative of :

8

$$y = \tan^{-1} x.$$

(c) If

$$z = \tan^{-1} \frac{x^3 + y^3}{x - y},$$

prove that :

8

$$x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} = \sin 2z.$$

Section IV

4. (a) Show that straight line :

$$\frac{x}{a} + \frac{y}{b} = 1$$

touches the curve :

$$y = be^{-x/a}$$

at the point, where the curve crosses the axis of

y .

8

- (b) Examine the following function for maximum and minimum

value : 8

$$f(x) = x^5 - 5x^4 + 5x^3 - 1$$

- (c) Trace the curve : 8

$$x(x^2 + y^2) = a(x^2 - y^2)$$

Section V

5. (a) Show that the maximum value of : 8

$$\left(\frac{1}{x}\right)^x \text{ is } e^{1/e}.$$

- (b) Prove that : 8

$$x - \frac{x^3}{3!} < \sin x < x - \frac{x^3}{3!} + \frac{x^5}{5!}, \quad 0 < x < \pi.$$

- (c) Show that : 8

$$\lim_{n \rightarrow 0} \left(\frac{1}{x^2} - \frac{1}{\sin^2 x} \right) = -\frac{1}{3}.$$

Section VI

6. (a) Evaluate :

9

$$\int_0^{\frac{\pi}{2}} \frac{(\sin x)^{3/2}}{(\sin x)^{3/2} + (\cos x)^{3/2}} dx$$

(b) Find the area bounded by the curve :

9

$$xy^2 = a^2 (a - x) \text{ and } y\text{-axis.}$$

(c) Find the arc length of the curve :

9

$$x = a(\theta + \sin \theta), y = a(1 - \cos \theta)$$