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Your Roll No.....

7626

B.A. (Prog.)/I **D-II**

MATHEMATICS—Paper I

(Algebra and Calculus)

(New Course : Admissions of 2006 and onwards)

Time : 3 *Hours* *Maximum Marks* : 100

(Write your Roll No. on the top immediately on receipt of this question paper.)

Note :— The maximum marks printed on the question paper are applicable for the students of Category 'B'. These marks will, however, be scaled down proportionately in respect of the students of regular colleges, Category 'A' at the time of posting of awards for compilation of result.

Attempt any *two* parts from each section.

P.T.O.

PART A

(ALGEBRA)

SECTION-I

1. (*a*) Prove that set of vectors :

$$\{(2, 2, -3), (0, -4, 1), (3, 1, -4)\}$$

in $\mathbf{R}^3$ is linearly dependent. 9

(*b*) Determine the characteristic roots and the corresponding characteristics vectors of the matrix : 9

$$\begin{bmatrix} 3 & 15 & 10 \\ 0 & 3 & 11 \\ 0 & 0 & 3 \end{bmatrix}.$$

(*c*) Using matrices solve the following system of equations : 9

$$x + 2y - z = 3$$

$$3x - y + 2z = 1$$

$$2x - 2y + 3z = 2$$

$$x - y + z = -1$$

SECTION-II

2. (*a*) Prove that :

$$(1 + \cos\alpha + i\sin\alpha)^p + (1 + \cos\alpha - i\sin\alpha)^p$$

$$= 2^{p+1} \cos^p(\alpha/2) \cos^p(p\alpha/2). \quad 8$$

(*b*) Prove that :

$$1 + \cos 9\theta = (1 + \cos\theta)(16\cos^4\theta - 8\cos^3\theta -$$

$$12\cos^2\theta + 4\cos\theta + 1)^2 \quad 8$$

(*c*) If α, β, γ be the roots (all non-zero) of the equation

$$x^3 - 3x^2 + 5x - 7 = 0,$$

find the value of $\sum\frac{\alpha}{\beta}$ and $\sum\alpha^2$. 8

P.T.O.

PART B

(CALCULUS)

SECTION-III

3. (*a*) Prove that if a function f is derivable at a point, it is continuous at that point. Show by an example that the converse is not true. 8

(*b*) If

$$v = r^m,$$

where

$$r^2 = x^2 + y^2 + z^2,$$

prove that

$$\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} = m(m + 1)r^{m-2}.$$ 8

(*c*) If

$$y = \frac{\sin^{-1} x}{\sqrt{1 - x^2}},$$

prove that :

$$(1 - x^2)y_{n+2} - (2n + 3)xy_{n+1} - (n + 1)^2 y_n. \quad 8$$

SECTION-IV

4. (*a*) Prove that the equation of the normal to the asteroid :

$$x^{2/3} + y^{2/2} = b^{2/3}$$

may be written in the form : 8

$$x\cos\varphi - y\sin\varphi + b\cos 2\varphi = 0.$$

(*b*) Find the asymptotes of the curve : 8

$$y^2x + yx^2 + xy + x^2 + 4y = 0.$$

(*c*) Trace the curve : 8

$$y^2(2a - x) = x^3.$$

SECTION-V

5. (*a*) State and prove Lagrange's Mean value theorem. 8

(*b*) Prove that total surface area of a right circular cylinder with a given volume is minimum when its height is equal to its diameter. 8

(*c*) Evaluate :

$$\lim_{n\to 0}\left(\frac{(1+x)^{1/x}-e+\frac{ex}{2}}{x^2}\right)$$ 8

SECTION-VI

6. (*a*) Evaluate :

$$\int \frac{dx}{(x^2-1)\sqrt{x^2+1}}$$ 9

(*b*) Find the area of the loop of the curve :

$$a^2y^2 = a^2(a - x)\ (3a - x)$$ 9

(*c*) Find the arc length of the portions of the parabola $y^2 = 20x$ cut-off by its latus rectum. 9