[This question paper contains 4 printed pages.]

5511 *Your Roll No. .....*

B.A. Prog. / I **D**

MATHEMATICS : Paper I

(Algebra and Calculus)

(Admissions of 2004 and onwards)

Time : 3 Hours *Maximum Marks : 75*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Note :– The maximum marks printed on the question paper are applicable for the students of the regular colleges (Cat. 'A'). These marks will, however, be scaled up proportionately in respect of the students of NCWEB at the time of posting of awards for compilation of result.

All the **six** *questions are compulsory and carry equal marks.*

Attempt any **two** *parts from each question.*

(a) Define subspace of a vector space over a field F. Prove that the intersection of two subspaces of a vector space V is a subspace of V.

P.T.O.

(b) Reduce the matrix A, where

$$A = \begin{bmatrix} 1 & 3 & 6 & -1 \\ 1 & 4 & 5 & 1 \\ 1 & 5 & 4 & 3 \end{bmatrix}$$

to its normal form and hence determine its rank.

(c) Solve completely the system of equations

$x + y + z + w = 0$

$x + 3y - 2z + w = 0$

$2x - 3z + 2w = 0$

$3x + 3y + 3w = 0$

2. (a) Express $\cos 7\theta$ and $\sin 7\theta$ in powers of $\cos\theta$ and $\sin\theta$. Hence obtain an expression for $\tan 7\theta$ in powers of $\tan\theta$.

(b) Solve the equation

$$4x^4 + 8x^3 + 13x^2 + 2x + 3 = 0,$$

given that the sum of two of the roots is zero.

(c) Form an equation of the lowest degree with real coefficients which has $2+i$ and $1-i$ as two of the roots.

3. (a) Find the value of a if the function f given by

$$f(x) = \begin{cases} 2x-1, & x < 2 \\ a, & x = 2 \\ x+1, & x > 2 \end{cases}$$

is continuous at x = 2.

(b) If $z = \sin^{-1}\dfrac{x^2+y^2}{x+y}$, then prove that

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = \tan z$$

(c) If $y = \tan^{-1} x$, then show that

$$(1+x^2)y_{n+2} + (2n+1)xy_{n+1} + n(n+1)y_n = 0.$$

4. (a) Find the pedal equation of the curve $r = a(1+\cos\theta)$.

(b) Find the asymptotes of the curve

$$x^2y - xy^2 + xy + y^2 + x - y = 0.$$

(c) Trace the curve

$$y^2(a^2 - x^2) = x^4.$$

5. (a) Verify Langrange's Mean Value theorem on the interval [1,3] for the function $f(x) = x^2 - 3x - 1$.

(b) Find the minimum and maximum values of the function $f(x) = x^4 + 4x^3 - 2x^2 - 12x + 7$.

P.T.O.

(c) Find $\lim_{x \to 0}\left(\frac{1}{x} - \frac{1}{\sin x}\right)$.

6. (a) Show that

$$\int \cos^m x \cos nx\,dx = \frac{\cos^m x \sin nx}{m+n} + \frac{m}{m+n}\int \cos^{m-1} x \cos(n-1)x\,dx$$

(b) Find the area of the smaller portion enclosed by the curves $x^2 + y^2 = 9$ and $y^2 = 8x$.

(c) Find the volume of the solid generated by rotating the ellipse $4x^2 + y^2 = 4$ about the x-axis.