This question paper contains 4+2 printed pages]

Your Roll No. .....

6731

B.A./B.Sc. (Hons.)/I **D**

MATHEMATICS—Unit I

(Vector, Calculus and Geometry)

(Admissions of 2008 and before)

Time : 2 *Hours* *Maximum Marks* : 38

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *one* question from each Section.

Marks are indicated on the margin.

Section I

(*a*) Show that : 3½

$$\text{div}\left(\vec{A} \times \vec{B}\right) = \vec{B}.\,\text{curl}\,\vec{A} - \vec{A}.\,\text{curl}\,\vec{B}.$$

P.T.O.

(*b*) If :

$$\vec{r} = x\vec{i} + y\vec{j} + z\vec{k} \text{ and } r = |\vec{r}|$$

then show that : 3

$$\nabla r^n = nr^{n-2}\,\vec{r}.$$

(*c*) A particle moves on the curve :

$$x = e^{-t},\ y = 2\cos 3t,\ z = 2\sin 3t,$$

find the magnitude of its initial velocity and acceleration. 3

2. (*a*) Find the directional derivative of

$$\phi = x^2 yz + 4xz^2$$

at the point (1, –2, –1) in the direction 3½

$$4\vec{i} + 4\vec{j} - 7\vec{k}.$$

(*b*) Find the value of p so that the curl of the vector given by :

$$(pxy - z^3)\vec{i} + (p-2)x^2\vec{j} + (1-p)xz^2\vec{k}$$

is zero. 3

(*c*) If :

$$\vec{A} = 2xz^2\vec{i} - yz\vec{j} + 3xz^3\vec{k} \text{ and}$$

$$\phi = x^2 yz,$$

find curl $\left(\phi \vec{A}\right)$ 3

Section II

3. (*a*) Find the limiting points of the coaxal system of circles determined by the circles : 5

$$x^2 + y^2 - 10x - 8y + 5 = 0 \text{ and}$$

$$x^2 + y^2 + 10x + 2y = 0.$$

(*b*) Find the locus of the point of intersection of two normals to the parabola :

$$y^2 = 4ax$$

which are at right angles to one another. 4½

P.T.O.

4. (*a*) CP and CQ are the conjugate semi-diameters of the ellipse :

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

and the circles with CP and CQ as diameters intersect at R. Show that R lies on the curve : 4½

$$2(x^2 + y^2)^2 = a^2x^2 + b^2y^2.$$

(*b*) Show that the locus of the midpoints of normal chords of the rectangular hyperbola 5

$$x^2 - y^2 = a^2,$$

is :

$$(y^2 - x^2)^3 = 4a^2x^2y^2.$$

Section III

5. Trace the following conic giving all the essential details : 9½

$$x^2 + 4xy + y^2 - 2x + 2y + 4 = 0$$

6. Trace the conic :

$$14x^2 - 4xy + 11y^2 - 44x - 58y + 17 = 0$$

giving full details about the conic. 9½

Section IV

7. (*a*) Find the equation of the sphere that passes through the two given points (0, 3, 0) and (–2, –1, –4) and cuts orthogonally the two spheres : 5

$$x^2 + y^2 + z^2 + x - 3z + 2 = 0 \text{ and}$$

$$2(x^2 + y^2 + z^2) + x + 3y + 4 = 0.$$

(*b*) Show that the cone : 4½

$$ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2hxy = 0$$

possesses three mutually perpendicular tangent planes if :

$$bc + ca + ab = f^2 + g^2 + h^2.$$

P.T.O.

8. (*a*) The tangent plane at any point of the sphere : 5

$$x^2 + y^2 + z^2 = r^2$$

meets the coordinate axes A, B, C. Show that the locus of the coordinate planes through A, B, C is the surface :

$$x^{-2} + y^{-2} + z^{-2} = r^{-2}.$$

(*b*) Find the equation of the right circular cylinder whose guiding circle is : 4½

$$x^2 + y^2 + z^2 - 9 = 0,$$

$$x - y + z = 3.$$