[This question paper contains 5 printed pages.]

1437 *Your Roll No.* ....

B.A. (Prog.)/I **E-I**

MATHEMATICS–Paper I

(Algebra and Calculus)

(New Course : Admissions of 2006 and onwards)

Time : 3 Hours *Maximum Marks : 100*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Note :– The maximum marks printed on the question paper are applicable for the students of Category 'B'. These marks will, however, be scaled down proportionately in respect of the students of regular colleges, Category 'A' at the time of posting of awards for compilation of result.

Attempt any two parts from each question.

1. (a) Let $V = \{(a_1, a_2, a_3): a_1, a_2, a_3 \in R\}$ be the set of all 3-tuples of real numbers.

Define the addition and scalar multiplication in V as follows:

$$(a_1, a_2, a_3) + (b_1, b_2, b_3) = (a_1 + b_1, a_2 + b_2, a_3 + b_3),$$

$$\alpha\ (a_1, a_2, a_3) = (\alpha a_1, \alpha a_2, \alpha a_3),\ \alpha \in R$$

Show that V is a vector space over the field R. (8)

(b) Show that the vectors (a_1, a_2) and (b_1, b_2) in $C^{(2)}$ are linearly dependent if and only if $a_1b_2 = a_2b_1$. (8)

(c) State Cayley-Hamilton and verify the same for the matrix:

$$\begin{pmatrix} -1 & -2 & 3 \\ -2 & 1 & 1 \\ 4 & -5 & 2 \end{pmatrix}$$ (8)

2. (a) Prove the following identity

$$64 \cos^7\theta = \cos 7\theta + 7 \cos 5\theta + 21 \cos 3\theta + 35 \cos \theta.$$

(8½)

(b) Solve the equation

$$x^3 - 9x^2 + 23x - 15 = 0,$$

the roots being in A.P. (8½)

(c) Find the modulus and the argument of the complex number

$$\frac{(1+\cos\theta+i\sin\theta)^5}{(\cos\theta+i\sin\theta)^3}.$$ (8½)

3. (a) Prove that the set of all n^{th} roots of unity forms an abelian group with respect to multiplication of complex numbers. (8½)

(b) If f= (1,2,3) and θ= (2,3,4) compute $\theta^{-1}f\theta$, where f, $\theta \in S_4$. (8½)

(c) Prove that the set

$$M_2=\left\{\begin{bmatrix} a & b \\ c & d \end{bmatrix}: a,b,c,d \in R\right\}$$

is a non commutative ring with respect to addition and multiplication of 2 × 2 matrices. Does the ring has unity? Justify. (8½)

4. (a) Examine the function

$$f(x)=\begin{cases}0 & x<0\\ x & 0\le x\le 1\\ 1-x & x>1\end{cases}$$

for continuity at $x = 0$ and $x = 1$. (8)

(b) Is the function

$f(x)= x|x|$

differentiable at $x = 0$? Prove your claim. (8)

(c) If $y = e^{m \sin^{-1}x}$, show that

$(1-x^2)y_2 - xy_1 - m^2y = 0.$

hence prove that for any positive integer n,

$y_{n+2}(0) = (m^2 + n^2)\, y_n(0).$ (8)

5. (a) Find the asymptotes of the curve

$x^3 + y^3 - 3xy + x + y - 1 = 0.$ (8½)

(b) Show that origin is a double point of the curve

$x^4 + y^4 + 3(x^3 + y^3) + x^2 - 2y^2 = 0$

What is its nature? Find the tangents to the curve at the origin. (8½)

(c) Find the equation of the tangent to the curve

$$x^n + y^n = a^n$$

at the point (p, q). (8½)

(a) State Lagrange's mean value theorem. Use it to show that a strictly monotonically increasing function, $f(x)$ on $[a, b]$ satisfies

$f'(x) > 0$, $a < x < b$. (8½)

(b) Obtain Maclaurin's series expansion of $f(x) = e^{2x}$ (8½)

(c) Find the maxima and minima of the function

$$f(x) = \sin^2 x + \cos x, \ -\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$$ (8½)