[This question paper contains 4 printed pages.]

1401 *Your Roll No.* ....

B.A. (Prog.)/I **E-I**

MATHEMATICS–Paper I

(Algebra and Calculus)

(NC : Admissions of 2006 onwards)

Time : 3 Hours *Maximum Marks : 100*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any two parts from each question.

1. (a) Prove that the set $M_2 = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} : a, b, c, d \in R \right\}$ is a vector space over R with respect to matrix addition and multiplication of a matrix by a scalar. (8)

(b) Find the row rank and column rank of the matrix

$$\begin{pmatrix} 4 & 5 & 6 \\ 1 & 2 & 3 \\ 3 & 4 & 5 \end{pmatrix}$$

and show that they are equal. (8)

(c) For what values of λ and μ do the following system of equations

$x + y + z = 6,$

$x + 2y + 3z = 10,$

$x + 2y + \lambda z = \mu$

have (i) unique solution (ii) no solution (iii) an infinite number of solutions? (8)

2. (a) Find a necessary condition for the roots of the equation

$x^3 - px^2 + qx - r = 0$, to be in H.P. (8½)

(b) State and prove De Moivre's Theorem for integral indices. (8½)

(c) Express $\sin^6\theta$ in terms of cosines of multiples of θ. (8½)

3. (a) Compute $a^{-1}ba$, where

(i) a = (5 7 9), b = (1 2 3)

(ii) a = (1 3 4), (2 3 5 4) (8½)

(b) Let G = { (a, b): a, b∈ R both not zero} and '*' be a binary operation defined by (a, b) * (c, d) = (ac - bd , ad + bc) for all a, b, c, d ∈ R

Show that (G, *) is a commutative group. (8½)

(c) Prove that set **Z** of all integers is a ring with respect to the addition and multiplication of integers. (8½)

4. (a) If

$x = a(\cos\theta + \theta \sin \theta),\ y = a(\sin \theta - \theta \cos \theta),$

find $\frac{d^2y}{d^2x}$. (8)

(b) Discuss the kind of discontinuity, if any, of the function defined as:

$$f(x)=\begin{cases} \frac{x-|x|}{x} & when\ x \neq 0 \\ 2 & when\ x = 0 \end{cases}$$ (8)

(c) If

$y = e^{m \sin^{-1}x}$,

show that:

$(1 - x^2)\, y_{n+2} - (2n + 1)\, xy_{n+1} - (n^2 + m^2)\, y_n = 0.$

(8)

5. (a) Find the asymptotes of the curve:

$x^3 + 3x^2y - 4y^3 - x + y + 3 = 0$ (8½)

(b) The tangent at two points P, Q on the cycloid:

$x = a(\theta - \sin\theta),\ y = a(1 - \cos\theta)$

are at right angles, show that if r_1, r_2 be the radii of curvature at these points then:

$r_1^2 + r_2^2 = 16a^2.$ (8½)

(c) Trace the curve :

$y^2(a^2 + x^2) = x^2(a^2 - x^2).$ (8½)

6. (a) State Lagrange's mean value theorem. Verify it for the function f(x)=sin(x) in the interval [0, π/2]. (8½)

(b) Evaluate

$$\lim_{x\to 0} \frac{\sin x - x + \frac{x^3}{6}}{x^5}$$ (8½)

(c) Assuming the possibility of expansion, prove that:

$$\sin x = \frac{1}{\sqrt{2}}\left[1 + \left(x - \frac{\pi}{4}\right) - \frac{\left(x - \frac{\pi}{4}\right)^2}{2!} - \frac{\left(x - \frac{\pi}{4}\right)^3}{3!} \ldots\ldots\right].$$

(8½)

(600)