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Your Roll No. ....

B.A. Prog. / I

E

MATHEMATICS : Paper I

(Algebra and Calculus)

(Admissions of 2004 and onwards)

Time : 3 Hours

Maximum Marks : 75

*(Write your Roll No. on the top immediately  
on receipt of this question paper.)*

*Note :- The maximum marks printed on the question paper are applicable for the students of the regular colleges (Cat. A). These marks will, however, be scaled up proportionately in respect of the students of NCWEB/SOL at the time of posting of awards for compilation of result.*

*Attempt any two parts from each question.*

**PART - A (ALGEBRA)**

**SECTION-I**

1. (a) Prove that set of vectors :

$\{(2, 2, -3), (0, -4, 1), (3, 1, -4)\}$  in  $\mathbb{R}^3$  is linearly dependent. (6.5)

P.T.O.

(b) Reduce the matrix :

$$\begin{bmatrix} 1 & 3 & 6 & -1 \\ 1 & 4 & 5 & 1 \\ 1 & 5 & 4 & 3 \end{bmatrix}$$

to its normal form and hence find its rank. (6.5)

(c) Determine the characteristics roots and the corresponding characteristics vectors of the matrix :

$$\begin{bmatrix} 3 & 2 & 0 \\ 0 & 5 & 1 \\ 0 & 0 & 2 \end{bmatrix} \quad (6.5)$$

## SECTION-II

2. (a) Find the sum of series :

$$\cos \theta + x \cos 2\theta + x^2 \cos 3\theta \dots \dots \dots \text{up to } n \text{ terms} \quad (6.5)$$

(b) If  $z = \cos \theta + i \sin \theta$ , show that :

$$\frac{z^{2n} - 1}{z^{2n} + 1} = i \tan n\theta \quad (6.5)$$

(c) If  $\alpha, \beta, \gamma$  are the roots of the equation :

$$x^3 - 3x^2 + 6x - 2 = 0$$

from the equation whose roots are

$$\beta^2 + \gamma^2, \gamma^2 + \alpha^2, \alpha^2 + \beta^2. \quad (6.5)$$

## PART - B (CALCULUS)

## SECTION-III

3. (a) Prove that if a function  $f$  is derivable at a point, it is continuous at that point. Show by an example that the converse is not true. (6)

(b) If  $y = e^{m \cos^{-1} x}$ , Show that :

$$(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - (n^2 - m^2)y_n = 0 \quad (6)$$

- (c) State and prove Euler's theorem on homogeneous function. If

$u = \log \left( \frac{x^2 + y^2}{x + y} \right)$ , use this theorem to show that

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 1 \quad (6)$$

## SECTION-IV

4. (a) Prove that the equation of the normal to the asteroid :

$$x^{2/3} + y^{2/3} = a^{2/3}$$

may be written in the form

$$x \cos \phi - y \sin \phi + a \cos 2\phi \quad (6)$$

- (b) Find the asymptotes of the curve :

$$x^3 + x^2y - xy^2 - y^3 - 2x^2 + 2y^2 + x + y + 1 = 0 \quad (6)$$

P.T.O.

- (c) Find the position and nature of the double point on the curve :

$$x^3 - y^2 - 7x^2 + 4y + 15x - 13 = 0 \quad (6)$$

### SECTION-V

5. (a) State and Prove Lagrange's Mean value Theorem. (6)
- (b) Find the maximum and minimum value of the function :

$$f(x) = x + \frac{1}{x}, \quad x \neq 0 \quad (6)$$

- (c) Evaluate

(i)  $\lim_{x \rightarrow 0} (\sin x)^x$

(ii)  $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x \sin x}$  (6)

### SECTION-VI

6. (a) Evaluate :

$$\int \frac{dx}{(x^2 - 1)\sqrt{x^2 + 1}} \quad (6.5)$$

- (b) Find the volume of a sphere of radius  $a$  using integration. (6.5)
- (c) Find the surface area of the solid of revolution, obtained by revolving the ellipse :

$$\frac{x^2}{4} + \frac{y^2}{9} = 1$$

about X-axis.

(6.5)

(500)