This question paper contains 4 printed pages.

334 *Your Roll No.* ....

B.A. (Prog.) / I **E**

MATHEMATICS : PAPER I

(Algebra and Calculus)

(Admissions of 2004 and onwards)

Time : 3 hours *Maximum Marks* : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

NOTE:— *The maximum marks printed on the question paper are applicable for the students of the regular colleges (Cat. A). These marks will, however, be scaled up proportionately in respect of the students of NCWEB at the time of posting of awards for compilation of result.*

All the six questions are compulsory.

Attempt any two parts from each question.

1. (a) Show that the vector (1, 2, 1), (3, 1, 5), (3, –4, 7) in $\mathbf{R}^{(3)}$ are linearly dependent. 6

   (b) Reduce the matrix to normal form and hence determine its rank:

P. T O.

$$\begin{pmatrix} 1 & 1 & 2 & 3 \\ 1 & 3 & 0 & 3 \\ 1 & -2 & -3 & -3 \\ 1 & 1 & 2 & 3 \end{pmatrix}$$ 6

(c) Find the characteristic equation of the matrix:

$$A=\begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 1 & 0 & -1 \end{pmatrix}$$

and hence compute its cube. 6

2. (a) If $m=\cos\alpha+i\ \sin\alpha$ and $n=\cos\beta+i\sin\beta$, prove that:—

$$\frac{m-n}{m+n}=i\ \tan\left(\frac{\alpha-\beta}{2}\right).$$ 61/2

(b) Prove that:

$$-128\ \sin^6\theta\cos^2\theta=\cos 8\theta-4\cos 6\theta+4\cos 4\theta+4\cos 2\theta-5$$

61/2

(c) If α, β, γ are the roots of $x^3+px^2+qx+r=0$ evaluate:

(i) $\sum\left(\frac{\alpha}{\beta}\right)$

(ii) $\sum\alpha^2\beta$ 61/2

3. (a) Discuss the continuity at $x=0, 1, 2$ of the function

$$f(x)=\begin{cases} -x^2 & \text{when } x\le 0 \\ 5x-4 & \text{when } 0<x\le 1 \\ 4x^2-3x & \text{when } 1\le x<2 \\ 3x+4 & \text{when } x\ge 2 \end{cases}$$ 6

(b) If $y=\sin(m\sin^{-1}\theta)$, show that:

$$(1-x^2)y_{n+2}-(2n+1)xy_{n+1}+(n^2-m^2)y_n=0.$$ 6

(c) If $u=\sin^{-1}\left(\frac{x^2+y^2}{x+y}\right)$, prove that:

$$x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\tan u.$$ 6

4. (a) Prove that the equation of the normal to the asteroid $x^{2/3}+y^{2/3}=a^{2/3}$ can be written in the form $x\sin\phi-y\sin\phi+a\cos 2\phi=0.$ 6

(b) For the cycloid $x=a(\theta+\sin\theta), y=a(1-\cos\theta)$, prove that ρ, radius of curvature, is:

$$\rho=4a\cos\frac{\theta}{2}.$$ 6

(c) Trace the curve

$$y^2(2a-x)=x^3,\ a>0.$$ 6

5. (a) State Lagrange's Mean Value Theorem. Verify Lagrange's Mean Value Theorem for the function

$f(x)=x(x-1)(x-2)$ in the interval $\left[0, \frac{1}{2}\right]$. 61/2

(b) Show that the maximum value of $\left(\frac{1}{x}\right)^x$ is $e^{1/e}$. 61/2

(c) Evaluate:

(i) $\lim_{x \to 0} (\cot)^{\frac{1}{\log x}}$

(ii) $\lim_{x \to \frac{\pi}{2}} \left(\frac{\tan 3x}{\tan x}\right)$. 61/2

6\. (a) Evaluate:

(i) $\int \frac{(1+x^2)}{(1+x^4)} dx$

(ii) $\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}} dx$ 61/2

(b) Find the whole area of the curve:

$$x^2(x^2+y^2)=a^2(x^2-y^2).$$ 61/2

(c) Find the volume of solid generated by rotating the ellipse $4x^2+y^2=4$ about x-axis. 61/2

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Attempt any **two** *parts from each question.*

PART – A (ALGEBRA)

SECTION–I

1. (a) Prove that set of vectors :

$\{(2, 2, -3), (0, -4, 1), (3, 1, -4)\}$ in R^3 is linearly dependent. (6.5)

(b) Reduce the matrix :

$$\begin{bmatrix} 1 & 3 & 6 & -1 \\ 1 & 4 & 5 & 1 \\ 1 & 5 & 4 & 3 \end{bmatrix}$$

to its normal form and hence find its rank. (6.5)

(c) Determine the characteristics roots and the corresponding characteristics vectors of the matrix :

$$\begin{bmatrix} 3 & 2 & 0 \\ 0 & 5 & 1 \\ 0 & 0 & 2 \end{bmatrix}$$ (6.5)

SECTION–II

2. (a) Find the sum of series :

$\cos\theta + x \cos 2\theta + x^2 \cos 3\theta$ .... up to n terms (6.5)

(b) If $z = \cos\theta + i \sin\theta$, show that :

$$\frac{z^{2n} - 1}{z^{2n} + 1} = i \tan n\theta$$ (6.5)

(c) If α, β, γ are the roots of the equation :

$$x^3 - 3x^2 + 6x - 2 = 0$$

from the equation whose roots are

$\beta^2 + \gamma^2, \gamma^2 + \alpha^2, \alpha^2 + \beta^2$. (6.5)

PART – B (CALCULUS)

SECTION–III

3. (a) Prove that if a function f is derivable at a point, it is continuous at that point. Show by an example that the converse is not true. (6)

(b) If $y = e^{m\cos^{-1}x}$, Show that :

$$(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - (n^2 - m^2)y_n = 0$$

(6)

(c) State and prove Euler's theorem on homogeneous function. If

$u = \log\left(\frac{x^2 + y^2}{x + y}\right)$, use this theorem to show that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial x} = 1$$

(6)

SECTION–IV

4. (a) Prove that the equation of the normal to the asteroid :

$$x^{2/3} + y^{2/2} = a^{2/3}$$

may be written in the form

$x \cos \varphi - y \sin \varphi + a \cos 2\varphi$ (6)

(b) Find the asymptotes of the curve :

$x^3 + x^2y - xy^2 - y^3 - 2x^2 + 2y^2 + x + y + 1 = 0$ (6)

P.T.O.

(c) Find the position and nature of the double point on the curve :

$x^3 - y^2 - 7x^2 + 4y + 15x - 13 = 0$ (6)

SECTION–V

5. (a) State and Prove Lagrange's Mean value Theorem. (6)

(b) Find the maximum and minimum value of the function :

$$f(x) = x + \frac{1}{x},\ x \neq 0$$ (6)

(c) Evaluate

(i) $\lim_{x \to 0} (\sin x)^x$

(ii) $\lim_{x \to 0} \frac{1 - \cos x}{x \sin x}$ (6)

SECTION–VI

6. (a) Evaluate :

$$\int \frac{dx}{(x^2 - 1)\sqrt{x^2 + 1}}$$ (6.5)

(b) Find the volume of a sphere of radius a using integration. (6.5)

(c) Find the surface area of the solid of revolution, obtained by revolving the ellipse :

$$\frac{x^2}{4} + \frac{y^2}{9} = 1$$

about X-axis. (6.5)

(500)