[This question paper contains 2 printed pages.]

Sr. No. of Question Paper : 5176 **D** **Your Roll No...............**

Unique Paper Code : 235151

Name of the Course : **B.A. (Prog.) – I**

Name of the Paper : Mathematics : Calculus

Semester : I

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **two** parts from each question.

1. (a) Find

$$\lim_{x \to 0} \frac{1 - 2\cos x + \cos 2x}{x^2}$$ (6)

(b) Examine the continuity of the function f defined by

$$f(x) = \begin{cases} \frac{x^3 - 8}{x^2 - 4}, & \text{if } x \neq 2, -2 \\ 3 \quad , & \text{if } x = 2, -2 \end{cases}$$

at $x = 2, -2$. (6)

(c) Discuss the derivability of the following functions

$$f(x) = \begin{cases} 2x - 3, & 0 \leq x \leq 2 \\ x^2 - 3, & 2 < x \leq 4 \end{cases}$$

at $x = 2, 4$. (6)

2. (a) If y = acos(logx) + bsin(logx), Show that

$$x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$$ (6½)

(b) State Leibnitz's theorem. Find the n^{th} derivative of $y = x^4 e^{ax}$. (6½)

(c) If $z = x^2 \tan^{-1}\left(\frac{y}{x}\right) - y^2 \tan^{-1}\left(\frac{x}{y}\right)$, prove that

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{x^2 - y^2}{x^2 + y^2} \qquad (6½)$$

3. (a) Find the equation of the tangent and normal to the curve $y(x-2)(x-3) - x + 7 = 0$ at the point where it meets the x-axis. (6)

(b) Show that the length of the portion of the tangent to the curve $x = a\cos^3\theta$, $y = a\sin^3\theta$ intercepted between the co-ordinate axis is constant. (6)

(c) For the cycloid $x = a(\theta + \sin\theta)$, $y = a(1 - \cos\theta)$, prove that radius of curvature

$$r = 4a\cos\frac{\theta}{2} \qquad (6)$$

4. (a) Determine the position and nature of the double points on the curve

$$x^3 - 2x^2 - y^2 + x + 4y - 4 = 0 \qquad (6½)$$

(b) Find the asymptotes of the curve

$$x^3 + 2x^2y - xy^2 - 2y^3 + xy - y^2 - 1 = 0 \qquad (6½)$$

(c) Trace the curve $y^2x = a^2(a-x)$. (6½)

5. (a) Discuss the applicability of Rolle's theorem in the interval [–3,0] to the function

$$f(x) = x(x+3)e^{-x/2} \qquad (6)$$

(b) State Lagrange's mean value theorem. Verify it for the function $f(x) = (x-1)(x-2)(x-3)$ in the interval [1,4]. (6)

(c) Find Maclaurin's power series expansion of the function sin(ax) in ascending powers of x. (6)

6. (a) Show that the maximum value of $\left(\frac{1}{x}\right)^x$ is $(e)^{1/e}$. (6½)

(b) Evaluate $\lim_{x\to 0} \frac{\sin x - x + \frac{1}{6}x^3}{x^5}$. (6½)

(c) Separate the intervals in which the function $f(x) = 2x^3 - 9x^2 + 12x - 5$. In increasing or decreasing. (6½)

(700)