[This question paper contains 2 printed pages.]

Sr. No. of Question Paper : 5175 **D** **Your Roll No...............**

Unique Paper Code : 235151

Name of the Course : **B.A. (Prog.)**

Name of the Paper : Mathematics : Calculus

Semester : I

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **two** parts from each question.

1. (a) Show that

$$\lim_{x\to 0} \frac{xe^{1/x}}{1+e^{1/x}} = 0 \qquad (6)$$

(b) Obtain that points of discontinuity of the function f defined on [0,1] as follows

$$f(x) = 0,\ f(x) = \frac{1}{2} - x;\ \text{if } 0 < x < \frac{1}{2}$$

$$f\left(\frac{1}{2}\right) = \frac{1}{2},\ f(x) = \frac{1}{2} - x,\ \text{if } \frac{1}{2} < x < 1$$

$$f(1) = 1 \qquad (6)$$

(c) Show that the function $f(x) = x|x|$ is derivable at the origin. (6)

2. (a) If $x = 2\cos t - \cos 2t$, $y = 2\sin t - \sin 2t$, find the value of $\frac{d^2y}{dx^2}$ when $t = \frac{\pi}{2}$. (6½)

(b) If $= e^{\tan^{-1}x}$, then show that $(1 + x^2)y_{n+2} + [2(n + 1)x - 1]y_{n+1} + n(n + 1)y_n = 0$. (6½)

(c) Prove that if $z = \frac{x^2y^2}{x+y}$, then

$$x\frac{\partial^2 z}{\partial x^2} + y\frac{\partial^2 z}{\partial x \partial y} = 2\frac{\partial z}{\partial x} \qquad (6\tfrac{1}{2})$$

3. (a) Find the equations of tangent and the normal at (x_1, y_1) to the curve

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad (6)$$

(b) The tangent at any point of the curve $x = a\cos^3\theta$, $y = b\sin^3\theta$ meets the coordinate axes A and B respectively. Show that the locus of the point with (OA, OB) as coordinates is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad (6)$$

(c) Show that the radius of curvature at any point of the curve $x = a\cos^3\theta$, $y = a\sin^3\theta$ is equal to three times the length of the perpendicular from the origin to the tangent. (6)

4. (a) Find the asymptotes of the curve $y^3 + x^2y + 2xy^2 - y + 1 = 0$. (6½)

(b) Find the position and nature of the double points on the curve

$$x^4 - 2ay^3 - 3a^2y^2 - 2a^2x^2 + a^4 = 0 \quad (6½)$$

(c) Trace the curve $y^2(a^2 - x^2) = x^4$. (6½)

5. (a) State Rolle's theorem. Discuss the applicability of Rolle's theorem for

$$f(x) = e^x(\sin x - \cos x)$$

in the interval $[\pi/4, 5\pi/4]$. (6)

(b) State Cauchy's mean value theorem.

If in the Cauchy's mean value theorem, $f(x) = e^x$, $g(x) = e^{-x}$, Show that c is the arithmetic mean between a and b where $a < c < b$. (6)

(c) Show that $\frac{x}{1+x} < \log(1 + x) < x$ for all $x > 0$. (6)

6. (a) Assuming the possibility of expansion, expand $\tan^{-1}x$ as far as term containing x^5. (6½)

(b) Evaluate

$$\lim_{x\to 0} \frac{x\cos x - \log(1 + x)}{x^2} \quad (6½)$$

(c) Examine the following function for maximum and minimum values

$f(x) = x^5 - 5x^4 + 5x^3 - 1$ (6½)

(500)