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Roll No. | | | | | | | | | | |

S. No. of Question Paper : **76**

Unique Paper Code : **235151** **E**

Name of the Paper : **Calculus**

Name of the Course : **B.A. (Prog.)-I Mathematics**

Semester : **I**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any *two* parts from each question.

1. (*a*) Discuss the existence of the limit of the function :

$$f(x) = \begin{cases} 1 & , \text{ if } x \leq 1 \\ 2 - x & , \text{ if } 1 < x < 2 \\ 2 & , \text{ if } x \geq 2 \end{cases}$$

at $x = 1$ and $x = 2$. 6

(*b*) Discuss the kind of discontinuity, if any, of the function defined as :

$$f(x) = \begin{cases} \dfrac{x - |x|}{x} & \text{when } x \neq 0 \\ 2 & \text{when } x = 0 \end{cases}$$ 6

(*c*) Show that every differentiable function is continuous. Is the converse true ? Justify your answer. 6

P.T.O.

2. (*a*) If

$$x = a(\cos\theta + \theta\sin\theta),\ y = a(\sin\theta - \theta\cos\theta),$$

find $\frac{d^2y}{dx^2}$. 6½

(*b*) If

$$y = e^{m\sin^{-1}x},$$

show that :

$$\left(1 - x^2\right)y_{n+2} - (2n+1)xy_{n+1} - \left(n^2 + m^2\right)y_n = 0.$$ 6½

(*c*) If

$$u = \sin^{-1}\left(\frac{x^2 + y^2}{x + y}\right),$$

show that :

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \tan u.$$ 6½

3. (*a*) Show that the normal at any point of the curve :

$$x = a\cos\theta + a\theta\sin\theta,\ y = a\sin\theta + a\theta\cos\theta$$

is at a constant distance from the origin. 6

(*b*) Prove that the equation of the normal to the asteroid $x^{2/3} + y^{2/3} = a^{2/3}$ may be written in the form :

$$x\sin\phi - y\cos\phi + a\cos 2\phi = 0.$$ 6

(c) The tangents at two points P, Q on the cycloid :

$$x = a(\theta - \sin\theta),\ y = a(1 - \cos\theta)$$

are at right angles, show that if r_1, r_2 be the radii of curvature at these points then :

$$r_1^2 + r_2^2 = 16a^2. \qquad 6$$

4. (a) Find the asymptotes of the curve :

$$x^3 + 3x^2y - 4y^3 - x + y + 3 = 0. \qquad 6½$$

(b) Find the position and nature of the double points on the curve :

$$y^2 = (x - 1)(x - 2)^2. \qquad 6½$$

(c) Trace the curve :

$$y^2(a^2 + x^2) = x^2(a^2 - x^2). \qquad 6½$$

5. (a) State Rolle's theorem. Verify Rolle's theorem for the function :

$$f(x) = (x - a)^m (x - b)^n,\ x \in [a, b]$$

where m and n are positive integers. 6

(b) State Lagrange's mean the value theorem. Verify it for the function $f(x) = \sin(x)$ in the interval $[0, \pi/2]$. 6

(c) Assuming the possibility of expansion, prove that :

$$\sin x = \frac{1}{\sqrt{2}}\left[1 + \left(x - \frac{\pi}{4}\right) - \frac{\left(x - \frac{\pi}{4}\right)^2}{2!} - \frac{\left(x - \frac{\pi}{4}\right)^3}{3!} \ldots\ldots\right]. \qquad 6$$

6. (*a*) Separate the intervals in which the function :

$$f(x) = 2x^3 - 9x^2 + 12x - 5$$

is increasing or decreasing. 6½

(*b*) Evaluate :

$$\lim_{x \to 0} \frac{\sin x - x + \frac{1}{6}x^3}{x^5}.$$ 6½

(*c*) Investigate the maxima and minima of the function :

$$f(x) = (x - 1)^2 (x + 1).$$ 6½