

This question paper contains 4 printed pages]

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S. No. of Question Paper : 75

Unique Paper Code : 235151

E

Name of the Paper : Calculus

Name of the Course : B.A. (Prog.)-I Mathematics

Semester : I

Duration : 3 Hours

Maximum Marks : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any two parts from each question.

1. (a) Discuss the existence of the limit of the function :

$$f(x) = \begin{cases} -x^2 & , \text{ if } x \leq 0 \\ 5x - 4 & , \text{ if } 0 < x \leq 1 \\ 4x^2 - 3x & , \text{ if } 1 < x < 2 \\ 3x + 4 & , \text{ if } x \geq 2 \end{cases}$$

at the points $x = 0, 1, 2$.

6

(b) Examine the continuity of the function f defined by :

$$f(x) = \begin{cases} \frac{e^{1/x} - e^{-1/x}}{e^{1/x} + e^{-1/x}} & , \text{ } x \neq 0 \\ 0 & , \text{ } x = 0 \end{cases}$$

at $x = 0$. Also discuss the kind of discontinuity, if any.

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P.T.O.

(c) A function $f(x)$ is defined as follows :

$$f(x) = \begin{cases} 5x - 4 & , \quad 0 \leq x \leq 1 \\ 4x^2 - 3x & , \quad 1 < x < 2 \\ 3x + 4 & , \quad x \geq 2 \end{cases}$$

Discuss the differentiability of $f(x)$ at $x = 1$ and $x = 2$.

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2. (a) If $y = Ae^{px} + Be^{qx}$, show that :

$$\frac{d^2y}{dx^2} - (p + q)\frac{dy}{dx} + pqy = 0. \quad 6\frac{1}{2}$$

(b) If $y = \sin^{-1} x$, then show that $(1 - x^2)y''_2 - xy'_1 = 0$. Further show that :

$$(1 - x^2)y''_{n+2} - (2n + 1)xy'_{n+1} - n^2y_n = 0. \quad 6\frac{1}{2}$$

(c) State Euler's theorem. Verify Euler's theorem for the function :

$$z = x^n \log\left(\frac{y}{x}\right). \quad 6\frac{1}{2}$$

3. (a) Prove that the sum of the intercepts on the coordinate axes of any tangent to the curve

$$\sqrt{x} + \sqrt{y} = \sqrt{a} \text{ is constant.} \quad 6$$

(b) Find the condition for the curves :

$$ax^2 + by^2 = 1, \quad a'x^2 + b'y^2 = 1$$

to intersect orthogonally.

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- (c) Show that the radius of curvature at a point of the curve :

$$x = ae^{\theta}(\sin \theta - \cos \theta), y = ae^{\theta}(\sin \theta + \cos \theta)$$

is twice the distance of the tangent at the point from origin.

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4. (a) Find the asymptotes of the curve :

$$x^3 + x^2y - xy^2 - y^3 + 2xy + 2y^2 - 3x + y = 0.$$

6½

- (b) Determine the position and nature of the double points on the curve :

$$x^3 - y^2 - 7x^2 + 4y + 15x - 13 = 0.$$

6½

- (c) Trace the curve :

$$x(x^2 + y^2) = a(x^2 - y^2).$$

6½

5. (a) State Rolle's theorem. Discuss the applicability of Rolle's theorem for the function :

$$f(x) = 2 + (x - 1)^{2/3}$$

in the interval $[0, 2]$.

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- (b) State Cauchy's mean value theorem. Verify the Cauchy's mean value theorem for the

functions $f(x) = \sin x$, $g(x) = \cos x$, in the interval $\left[\frac{-\pi}{2}, 0\right]$.

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(c) Assuming the possibility of expansion, prove that :

$$\sin x = 1 - \frac{\left(x - \frac{\pi}{2}\right)^2}{2!} + \frac{\left(x - \frac{\pi}{2}\right)^4}{4!} - \dots$$

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6. (a) Show that :

$$\frac{u - v}{1 + v^2} < \tan^{-1} v - \tan^{-1} u < \frac{v - u}{1 + u^2},$$

$0 < u < v$ and deduce that :

$$\frac{\pi}{4} + \frac{3}{25} < \tan^{-1} \frac{4}{3} < \frac{\pi}{4} + \frac{1}{6}$$

6½

(b) Evaluate :

$$\lim_{x \rightarrow 0} \frac{e^x + e^{-x} - 2 \log(1 + x)}{x \sin x}$$

6½

(c) Examine the following function for maximum and minimum values :

$$f(x) = x^5 - 5x^4 + 5x^3 - 1.$$

6½