This question paper contains 4+2 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : 77

Unique Paper Code : **237151** **E**

Name of the Paper : **Basic Statistics and Probability**

Name of the Course : **B.A. (Programme) Statistics**

Semester : **I**

Duration : **3 Hours** Maximum Marks : **75**

*(Write your Roll No. on the top immediately on receipt of this question paper.)*

Question No. **1** is compulsory.

Attempt *six* questions in all.

Simple calculator can be used.

1. (*a*) Fill in the blanks : 5

(*i*) For a symmetrical distribution $\beta_1$ = ..................... .

(*ii*) Probability of impossible event = ..................... .

(*iii*) If A and B are mutually exclusive events, then $P(A \cup B)$ = ..................... .

(*iv*) Mean deviation is least when taken about ...................... .

(*v*) If one of the regression coefficients is greater than unity, then other must be ..................... .

(*b*) A random variable X has the following probability function : 5

| X | P(X) |
|---|---|
| 0 | 0 |
| 1 | $k$ |
| 2 | $2k$ |
| 3 | $2k$ |
| 4 | $3k$ |
| 5 | $k^2$ |
| 6 | $2k^2$ |
| 7 | $7k^2 + k$ |

(*i*) Find $k$

(*ii*) $P(0 < X < 5)$.

(*c*) The two regression equations are given to be :

$$8X - 10Y + 66 = 0, \; 40X - 18Y = 214$$

with variance of X = 9. Find :

(*i*) Mean values of X and Y.

(*ii*) The correlation coefficient between X and Y.

(*iii*) Standard Deviation of Y. 5

2. (*a*) Calculate the mean and standard deviation for the following table, giving the age distribution of 542 members : 6

| **Age** (in Years) | **No. of Members** |
|---|---|
| 20—30 | 3 |
| 30—40 | 61 |
| 40—50 | 132 |
| 50—60 | 153 |
| 60—70 | 140 |
| 70—80 | 51 |
| 80—90 | 2 |

(*b*) In a series of measurements, we obtain $m_1$ values of magnitude $x_1$, $m_2$ values of magnitude $x_2$ and so on. If $\bar{X}$ is the mean value of all the measurements, prove that the standard deviation is :

$$\sqrt{\frac{\sum m_r (k - x_r)^2}{\sum m_r} - \delta^2}$$

where $\bar{X} = k + \delta$ and $k$ is any constant. 6

3. (*a*) Let $r$ be the range and $s$ be the standard deviation of a set of observations $x_1, x_2, x_3$ ........, $x_n$, then prove that $s \le r$. 6

(*b*) What do you mean by Skewness and Kurtosis ? Prove that Kurtosis is greater than unity. 6

4. (*a*) Show that for $n$ events $A_1, A_2, \ldots\ldots, A_n$ : 6

(*i*) $P\left(\bigcap_{i=1}^{n} A_i\right) \ge \sum_{i=1}^{n} P(A_i) - (n-1)$

(*ii*) $P\left(\bigcup_{i=1}^{n} A_i\right) \le \sum_{i=1}^{n} P(A_i)$.

(*b*) A problem in statistics is given to three students A, B and C whose chances of solving it are $\frac{1}{2}$, $\frac{3}{4}$ and $\frac{1}{4}$ respectively. What is the probability that the problem will be solved if all of them try independently ? 6

5. (*a*) Prove that for any two events A and B : 6

$$P(A \cap B) \leq P(A) \leq P(A \cup B) \leq P(A) + P(B).$$

(*b*) State and prove Bayes' theorem. 6

6. (*a*) Define Karl Pearsonian Correlation Coefficient and show that it is independent of change of origin and scale. 6

(*b*) X and Y are two random variables with variances $\sigma_x^2$ and $\sigma_y^2$ and $r$ is the coefficient of correlation between them. If

$$U = X + KY \text{ and } V = X + \frac{\sigma_x}{\sigma_y} Y,$$

find the value of K so that U and V are uncorrelated. 6

7. (*a*) Given that

$$X = 4Y + 5 \text{ and } Y = KX + 4,$$

are the two lines of regression of X on Y and Y on X, respectively, show that $0 < 4K < 1$. If $K = 1/16$, find the means of the two variables and coefficient of correlation between them. 6

(*b*) In the usual notation, prove that : 6

$$R_{1.23}^2 = \left(r_{12}^2 + r_{13}^2 - 2r_{12}r_{23}r_{31}\right) / \left(1 - r_{23}^2\right).$$