This question paper contains 4 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **75**

Unique Paper Code : **235151** **E**

Name of the Paper : **Calculus**

Name of the Course : **B.A. (Prog.)-I Mathematics**

Semester : **I**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any *two* parts from each question.

1. (*a*) Discuss the existence of the limit of the function :

$$f(x) = \begin{cases} -x^2 & , \quad \text{if } x \le 0 \\ 5x - 4 & , \quad \text{if } 0 < x \le 1 \\ 4x^2 - 3x & , \quad \text{if } 1 < x < 2 \\ 3x + 4 & , \quad \text{if } x \ge 2 \end{cases}$$

at the points $x = 0, 1, 2$. 6

(*b*) Examine the continuity of the function f defined by :

$$f(x) = \begin{cases} \dfrac{e^{1/x} - e^{-1/x}}{e^{1/x} + e^{-1/x}}, & x \ne 0 \\ 0 & x = 0 \end{cases}$$

at $x = 0$. Also discuss the kind of discontinuity, if any. 6

(*c*) A function $f(x)$ is defined as follows :

$$f(x) = \begin{cases} 5x - 4 & , \quad 0 \le x \le 1 \\ 4x^2 - 3x & , \quad 1 < x < 2 \\ 3x + 4 & , \quad x \ge 2 \end{cases}$$

Discuss the differentiability of $f(x)$ at $x = 1$ and $x = 2$. 6

2. (*a*) If $y = Ae^{px} + Be^{qx}$, show that :

$$\frac{d^2y}{dx^2} - (p+q)\frac{dy}{dx} + pqy = 0.$$ 6½

(*b*) If $y = \sin^{-1} x$, then show that $(1 - x^2)y_2 - xy_1 = 0$. Further show that :

$$(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - n^2y_n = 0.$$ 6½

(*c*) State Euler's theorem. Verify Euler's theorem for the function :

$$z = x^n \log\left(\frac{y}{x}\right).$$ 6½

3. (*a*) Prove that the sum of the intercepts on the coordinate axes of any tangent to the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$ is constant. 6

(*b*) Find the condition for the curves :

$$ax^2 + by^2 = 1, \quad a'x^2 + b'y^2 = 1$$

to intersect orthogonally. 6

(c) Show that the radius of curvature at a point of the curve :

$$x = ae^{\theta}(\sin\theta - \cos\theta),\ y = ae^{\theta}(\sin\theta + \cos\theta)$$

is twice the distance of the tangent at the point from origin. 6

4. (a) Find the asymptotes of the curve :

$$x^3 + x^2y - xy^2 - y^3 + 2xy + 2y^2 - 3x + y = 0.$$ 6½

(b) Determine the position and nature of the double points on the curve :

$$x^3 - y^2 - 7x^2 + 4y + 15x - 13 = 0.$$ 6½

(c) Trace the curve :

$$x(x^2 + y^2) = a(x^2 - y^2).$$ 6½

5. (a) State Rolle's theorem. Discuss the applicability of Rolle's theorem for the function :

$$f(x) = 2 + (x - 1)^{2/3}$$

in the interval [0, 2]. 6

(b) State Cauchy's mean value theorem. Verify the Cauchy's mean value theorem for the functions $f(x) = \sin x$, $g(x) = \cos x$, in the interval $\left[\frac{-\pi}{2}, 0\right]$. 6

(c) Assuming the possibility of expansion, prove that : 6

$$\sin x = 1 - \frac{\left(x - \frac{\pi}{2}\right)^2}{2!} + \frac{\left(x - \frac{\pi}{2}\right)^4}{4!} - \ldots\ldots$$

6. (a) Show that :

$$\frac{u - v}{1 + v^2} < \tan^{-1} v - \tan^{-1} u < \frac{v - u}{1 + u^2},$$

$0 < u < v$ and deduce that : 6½

$$\frac{\pi}{4} + \frac{3}{25} < \tan^{-1}\frac{4}{3} < \frac{\pi}{4} + \frac{1}{6}.$$

(b) Evaluate : 6½

$$\lim_{x \to 0} \frac{e^x + e^{-x} - 2\log(1 + x)}{x \sin x}.$$

(c) Examine the following function for maximum and minimum values : 6½

$$f(x) = x^5 - 5x^4 + 5x^3 - 1.$$