This question paper contains 3 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **6663**

Unique Paper Code : **62351101** **FC**

Name of the Paper : **Calculus**

Name of the Course : **B.A. (Prog.) Mathematics**

Semester : **I**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any *two* parts from each question.

1. (*a*) Discuss the existence of the limit of the function :

$$f(x) = \begin{cases} x & x \leq 1 \\ 2 - x & 1 < x \leq 2 \\ -2 + 3x - x^2 & x > 2 \end{cases}$$

at $x = 1$ and $x = 2$. 6

(*b*) Find a value for the constant k, if possible, that makes the function f continuous everywhere, where f is defined by :

$$f(x) = \begin{cases} 7x - 2 & x \leq 1 \\ kx^2 & x > 1 \end{cases}$$ 6

(*c*) Prove that the function f defined by

$$f(x) = \begin{cases} \dfrac{x}{e^{\frac{1}{x}} + 1} & x \neq 0 \\ 0 & x = 0 \end{cases}$$

is not differentiable at origin. 6

2. (*a*) Find the *n*th derivative of $\sin^2 x \cos^3 x$. 6

(*b*) If $y = \sin^{-1} x$, then

$$(1 - x^2)y_{n+2} - (2n + 1)x\, y_{n+1} - n^2 y_n = 0.$$ 6

(*c*) If

$$z = \tan^{-1} \frac{x^3 + y^3}{x - y},$$

then using Euler's theorem, prove that :

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = \sin 2z.$$ 6

3. (*a*) Find the equation of tangent to the parabola $y^2 = 4x + 5$, which is parallel to the line $y - 2x + 3 = 0$. 6½

(*b*) Show that the length of the portion of the tangent to the curve $x = a\cos^3\theta, y = a\sin^3\theta$ intercepted between coordinate axis is constant. 6½

(*c*) If ρ_1, ρ_2 be the radius of curvature at the extremities of any chord of the cardiode $r = a(1 + \cos\theta)$ which passes through the pole, then show that $\rho_1^2 + \rho_2^2 = 16a^2/9$. 6½

4. (*a*) Find all the asymptotes of the curve :

$$x^2y - xy^2 + xy + y^2 + x - y = 0.$$ 6½

(*b*) Determine the position and nature of the double points on the curve :

$$x^3 - y^2 - 7x^2 + 4y + 15x - 13 = 0.$$ 6½

(*c*) Trace the curve :

$$y^2(a + x) = x^2(3a - x).$$ 6½

5. (*a*) State Euler's theorem and verify Rolle's theorem for the function

$$f(x) = (x - a)^m (x - b)^n, \quad x \in [a, b]$$

where m and n are positive integers. 6

(*b*) Prove that :

$$\frac{\tan x}{x} > \frac{x}{\sin x}, \quad 0 < x < \frac{\pi}{2}.$$ 6

(*c*) Obtain the Maclaurin's infinite series expansion of e^x, $x \in \mathbf{R}$. 6

6. (*a*) Find the maximum and minimum values of $(1 - x^2)e^x$. 6½

(*b*) Determine the values of p and q for which :

$$\lim_{x \to 0} \frac{x(1 + p\cos x) - q\cos x}{x^3}$$

exists and is finite. 6½

(*c*) Evaluate :

$$\lim_{x \to 0} (1 + x)^{\frac{1}{x}}.$$ 6½