This question paper contains 3 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **6664**

Unique Paper Code : **62351101** **FC**

Name of the Paper : **Calculus**

Name of the Course : **B.A. (Prog.) Mathematics**

Semester : **I**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any *two* parts from each question.

1. (*a*) Prove that :

$$\lim_{x \to 0} \frac{x - |x|}{x}$$

does not exist. 6

(*b*) Examine the continuity of the function at $x = 0$ and $x = 1$ for

$$f(x) = \begin{cases} -x^2 & x \le 0 \\ 5x - 4 & 0 < x \le 1 \\ x^2 - 3x & x > 1 \end{cases}$$

Also state the kind of discontinuity, if any. 6

(*c*) Prove that every differentiable function is continuous. Is the converse true ? Justify your answer. 6

P.T.O.

2. (*a*) Find the *n*th derivative of $\frac{x^2 + 4x + 1}{x^3 + 2x^2 - x - 2}$. 6

(*b*) If $y = e^{m \cos^{-1} x}$, then

$$\left(1 - x^2\right) y_{n+2} - (2n + 1) x\, y_{n+1} - \left(n^2 + m^2\right) y_n = 0.$$ 6

(*c*) State Euler's theorem. Also verify Euler's theorem for :

$$z = \tan^{-1}\left(\frac{y}{x}\right).$$ 6

3. (*a*) Prove that the sum of the intercept on the coordinate axis of the tangent to the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$ is constant. 6½

(*b*) Find the equations of the tangents and the normal at the point $\theta = \pi/2$ of the curve :

$$x = a(\theta + \sin\theta),\ y = a(1 + \cos\theta).$$ 6½

(*c*) Show that the curvature of a circle of radius a is $1/a$. 6½

4. (*a*) Find all the asymptotes of the curve :

$$x^3 + 2x^2y - xy^2 - 2y^3 + 2xy + 4y^2 + y - 1 = 0.$$ 6½

(*b*) Find the position and nature of the double points on the curve :

$$x^3 + 2x^2 + 2xy - y^2 + 5x - 2y = 0.$$ 6½

(*c*) Trace the curve :

$$y^2(a + x) = (a - x)x^2.$$ 6½

5. (*a*) State Rolle's theorem. Give the geometrical interpretation of Rolle's theorem and verify Rolle's theorem for the function $f(x) = (x - 2)(x + 1),\ x \in [-1, 2]$. 6

(b) Show that the function :

$$f(x) = 3x^3 - 9x^2 + 9x + 7$$

is strictly increasing everywhere. 6

(c) Prove that :

$$\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \text{.......},$$

for $-1 < x < 0$. 6

6. (a) State and prove Lagrange's Mean Value theorem. 6½

(b) Find the maximum value of $\frac{1}{x^x}$. 6½

(c) Evaluate :

$$\lim_{x \to \frac{\pi}{2}} (\sin x)^{\tan x}.$$ 6½