[This question paper contains 2 printed pages.]

Sr. No. of Question Paper : 5150 **F** **Your Roll No................**

Unique Paper Code : 235151

Name of the Paper : MT – Calculus

Name of the Course : **B.A. (Programme) – Mathematics**

Semester : I

Duration : 3 Hours

Maximum Marks : 45

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** questions are compulsory and carry equal marks.
3. Attempt any **TWO** parts from each question.

1. (a) Examine the continuity at x = 0, x = 1 of the function f defined as follows :

$$f(x) = \begin{cases} 2x^2 + 1 & \text{if } x \le 0 \\ x + 4 & \text{if } 0 < x \le 1 \\ 4x^3 + 1 & \text{if } x > 1 \end{cases}$$

Also discuss the kind of discontinuities, if any.

(b) Show that $\lim_{x \to 0} \frac{1 - \cos x}{x^2} = \frac{1}{2}$.

(c) Determine the values of a and b for which the function defined as

$$f(x) = \begin{cases} ax + b & \text{if } x \le 0 \\ 1 + x^2 & \text{if } x > 0 \end{cases}$$

is continuous.

2. (a) If $y = e^{m \sin^{-1} x}$, show that

$$(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - (n^2 + m^2)y_n = 0$$

(b) If $x = a \sin\theta$, $y = a(1 - \cos\theta)$, then find $\frac{d^2y}{dx^2}$ at $\theta = \pi$.

(c) If $u = \log\frac{x^2 + y^2}{x + y}$, then prove that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 1.$$

3. (a) Prove that the equation of the tangent at any point $(4m^2, 8m^3)$ of the curve $x^3 = y^2$ is $y = 3mx - 4m^3$ and show that it meets the curve again at $(m^2, -m^3)$, where it is normal if $9m^2 = 2$.

(b) Find the angle of intersection of the curves

$x^2 - y^2 = a^2$ and $x^2 + y^2 = a^2\sqrt{2}$.

(c) Find the radius of curvature for the curve $\sqrt{\frac{x}{a}} - \sqrt{\frac{y}{b}} = 1$ at the points where it touches the coordinate axes.

4. (a) Find all the asymptotes of the curve

$$(x^2 - y^2)(x + 2y) + 5(x^2 + y^2) + x + y = 0.$$

(b) Find the position and nature of the double points on the curve

$$x^4 - 4a x^3 + 2a y^3 + 4a^2 x^2 - 3a^2 y^2 - a^4 = 0.$$

(c) Trace the curve

$$x^2 y^2 = x^2 - 1.$$

5. (a) State Rolle's Theorem. Verify Rolle's theorem for the function

$$f(x) = (x - 1)^2 (x - 4)^3, \quad x \in [1, 4].$$

Also give geometrical interpretation of Rolle's theorem.

(b) Show that $\frac{2}{\pi} < \frac{\sin x}{x} < 1$, for $0 < x < \frac{\pi}{2}$.

(c) Obtain the Maclaurin's infinite series expansion of e^x, $x \in \mathbf{R}$.

6. (a) Find the maximum value of $\left(\frac{1}{x}\right)^x$.

(b) Use L' Hospital's rule to evaluate

$$\lim_{x\to 0} \frac{1 - \cos x^2}{x^2 \sin x^2}$$

(c) Evaluate $\lim_{x\to\frac{\pi}{2}} (\sin x)^{\tan x}$