[This question paper contains 2 printed pages.]

Sr. No. of Question Paper : 5151 **F** **Your Roll No...............**

Unique Paper Code : 235151

Name of the Paper : MT : Calculus

Name of the Course : **B.A. (Programme)**

Semester : I

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** questions are compulsory and carry equal marks.
3. Attempt any **TWO** parts from each question.

1. (a) Examine the continuity of the function

$$f(x) = \begin{cases} \dfrac{x-1}{1-e^{1/(x-1)}} & \text{if } x \neq 1 \\ 0 & \text{if } x = 1 \end{cases}$$

at x = 1.

(b) If $x^y = e^{x-y}$, then prove that

$$\frac{dy}{dx} = \frac{\log x}{(\log ex)^2}.$$

(c) Discuss the existence of limit of the function

$$f(x) = \begin{cases} x^2 & \text{if } x \leq 1 \\ 2-x^2 & \text{if } 1 < x < 2 \\ 0 & \text{if } x \geq 2 \end{cases}$$

at x = 1, 2.

2. (a) If $y = (\sin^{-1} x)^2$, then prove that

$$(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - n^2y_n = 0$$

(b) If $x = 2\cos t - 2\cos 2t$, $y = 2\sin t - \sin 2t$, then find the value of $\dfrac{d^2y}{dx^2}$ at $t = \dfrac{\pi}{2}$.

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(c) If $u = \sec^{-1}\left(\frac{x^2+y^2}{x+y}\right)$, then prove that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 2\cot u.$$

3. (a) If $p = x\cos\theta + y\sin\theta$ touches the curve

$$\left(\frac{x}{a}\right)^{\frac{n}{n-1}} + \left(\frac{y}{b}\right)^{\frac{n}{n-1}} = 1,$$

then show that $(a\cos\theta)^n + (b\cos\theta)^n = p^n$.

(b) Find the value of a for which the curves $y = 1 - ax^2$, $y = x^2$ are orthogonal.

(c) Prove that the radius of curvature of the curve $r^m = a^m\cos(m\theta)$, is given by

$$\rho = \frac{a^m}{(m+1)r^{m-1}}.$$

4. (a) Find the asymptotes of the curve

$$x^3 + x^2y - xy^2 - y^3 + 2xy + 2y^2 - 3x + y = 0.$$

(b) Find the position and nature of the double points on the curve

$$x^4 + y^3 - 2x^3 + 3y^2 - a^4 = 0$$

(c) Trace the curve

$$y^2 = (x-1)(x-2)(x-3).$$

5. (a) State Cauchy's Mean Value theorem. Give its geometrical interpretation. Also verify Cauchy's mean value theorem for the functions $f(x) = \sin x$, $g(x) = \cos x$ in the interval $\left[-\frac{\pi}{2}, 0\right]$.

(b) Show that the function $3x^3 - 9x^2 + 9x + 7$ is strictly increasing in every interval.

(c) Prove that $\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots$, for $0 \le x \le 1$.

6. (a) Find the maximum value of $\frac{\log x}{x}$ in $0 < x < \infty$.

(b) Find the maximum and minimum values of $\frac{1}{2}x - \sin x$ in $0 < x < \frac{\pi}{2}$.

(c) If $\lim_{x\to 0}\frac{\sin 3x - a\sin x}{x^3}$ is finite, then find the value of a and the limit.