[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 1962 **GC-3** **Your Roll No...............**

Unique Paper Code : 62351101

Name of the Paper : Calculus

Name of the Course : **B.A. (Prog.) Mathematics (CBCS)**

Semester : I

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **two parts** from each question.

1. (a) Discuss the existence of the limit of the function $f(x) = |2x - 1|$ at $x = \frac{1}{2}$. (6)

   (b) Examine the continuity of the function at x = 0 for

$$f(x) = \begin{cases} x\dfrac{e^{\frac{1}{x}} - 1}{e^{\frac{1}{x}} + 1} & x \neq 0 \\ 1 & x = 0 \end{cases}$$

   Also state the kind of discontinuity, if any. (6)

   (c) Examine the following function for differentiability at x = 0 and x = 1 :

$$f(x) = \begin{cases} x^2 & x \leq 0 \\ 1 & 0 < x \leq 1 \\ 1/x & x > 1 \end{cases}$$ (6)

2. (a) If $V = r^m$, where $r^2 = x^2 + y^2 + z^2$, show that

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = m(m+1)r^{m-2}.$$ (6)

(b) If $\cos^{-1}\left(\frac{y}{b}\right) = \log\left(\frac{x}{n}\right)^n$, prove that

$$x^2 y_{n+2} + (2n+1)x\,y_{n+1} + 2n^2 y_n = 0.$$ (6)

(c) If $z = \sec^{-1}\left(\frac{x^3+y^3}{x+y}\right)$, prove that

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = 2\cot z.$$ (6)

3. (a) If the tangent to the curve $\sqrt{\frac{x}{a}} + \sqrt{\frac{y}{b}} = 1$ cuts off intercept p and q from the axis of x & y respectively. Show that $\frac{p}{a} + \frac{q}{b} = 1$. (6)

(b) Find the point where the tangent to the curve $y = x^2 - 3x + 2$ is perpendicular to the line $y = x$. (6)

(c) Show that the radius of curvature for the curve

$x = a(\theta + \sin\theta)$, $y = a(1 - \cos\theta)$ is $4a\cos(\theta/2)$. (6)

4. (a) Find the asymptotes of the curve

$$x^3 + x^2y - xy^2 - y^3 + 2xy + 2y^2 - 3x + y = 0.$$ (6½)

(b) Find the position and nature of the double points on the curve

$$x^4 + y^3 - 2x^3 + 3y^2 - a^4 = 0$$ (6½)

(c) Trace the curve

$$x^2(x^2 + y^2) = 4(x^2 - y^2).$$ (6½)

5. (a) State and prove Lagrange's Mean Value theorem. (6)

(b) Separate the intervals in which the function

$$f(x) = 2x^3 - 15x^2 + 36x + 1$$

is increasing or decreasing. (6)

(c) Prove that $\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots$, for $0 \le x \le 1$. (6)

6. (a) State Cauchy's Mean Value theorem. Give its geometrical interpretation. Also verify Cauchy's mean value theorem for the functions $f(x) = \sin x$, $g(x) = \cos x$ in the interval $\left[-\frac{\pi}{2}, 0\right]$. (6½)

(b) Find the minimum and maximum value of the function x^x. (6½)

(c) If $\lim_{x \to 0} \frac{\sin 3x - a \sin x}{x^3}$ is finite, then find the value of a and the limit. (6½)