[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 1961 **GC-3** **Your Roll No................**

Unique Paper Code : 62351101

Name of the Paper : Calculus

Name of the Course : **B.A. (Prog.) Mathematics (CBCS)**

Semester : I

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **two parts** from each question.

1. (a) Discuss the existence of the limit of the function

$$f(x) = \begin{cases} x & x \le 1 \\ 2 - x & 1 < x \le 2 \\ -2 + 3x - x^2 & x > 2 \end{cases}$$

at x = 1 and x = 2. (6)

(b) A function f is defined by :

$$f(x) = \begin{cases} \dfrac{e^{\frac{1}{x}} - e^{-\frac{1}{x}}}{e^{\frac{1}{x}} + e^{-\frac{1}{x}}} & x \ne 0 \\ -1 & x = 0 \end{cases}$$

Examine f for continuity at x = 0. Also discuss the kind of discontinuity, if any. (6)

(c) Discuss the existence of the differentiability of the function

$$f(x) = |x| + |x-1| + |x-2|$$

at x = 0, 1 and 2. (6)

2. (a) If $u = x^2 \tan^{-1}\frac{y}{x} - y^2 \tan^{-1}\frac{x}{y}$, $x \neq 0$ and $y \neq 0$, then prove that

$$\frac{\partial^2 u}{\partial x \partial y} = \frac{x^2 - y^2}{x^2 + y^2}.$$ (6)

(b) If $y = \sin^{-1} x$, then

$$(1 - x^2)y_{n+2} - (2n + 1)x\, y_{n+1} - n^2 y_n = 0.$$ (6)

(c) State Euler's theorem and using it prove that, if $z = \log\left(\frac{x^4 + y^4}{x + y}\right)$, then

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = 3.$$ (6)

3. (a) Find the equation of the tangent to the curve $y^2 = 4x$ which makes an angle of 45° with the x – axis. (6½)

(b) Show that the normal at any point of the curve

$$x = a\cos\theta + a\theta\sin\theta,\ y = a\sin\theta - a\theta\cos\theta$$

is at a constant distance from the origin. (6½)

(c) If CP, CD be a pair of conjugate semi-diameter of the ellipse

$$\frac{x^2}{a^2} + \frac{x^2}{b^2} = 1$$

prove that the radius of curvature at P is $(CD)^3/ab$. (6½)

4. (a) Find the asymptotes of the curve

$$x^3 + 3x^2y - 4y^3 - x + y + 3 = 0.$$ (6½)

(b) Find the position and nature of the double points on the curve

$$x^3 + y^3 - 12x - 27y + 70 = 0.$$ (6½)

(c) Trace the curve

$$y^2 = (x - 2)(x - 3)(x - 4).$$ (6½)

5. (a) State Lagrange's Mean value theorem in the interval [a, a + h]. Prove that for any quadratic function $px^2 + qx + r$, the value of θ in Lagrange's theorem is always $\frac{1}{2}$ whatever p, q, r, a, h may be. (6)

(b) Show that $\frac{2}{\pi} < \frac{\sin x}{x} < 1$, for $0 < x < \frac{\pi}{2}$. (6)

(c) Obtain the Maclaurin's series expansion of cos x, $x \in R$. (6)

6. (a) State and prove Cauchy's Mean Value theorem. (6½)

(b) Find the maximum value of $\frac{\log x}{x}$ in $0 < x < \infty$. (6½)

(c) Evaluate $\lim_{x \to 0}(1+\sin x)^{\cot x}$. (6½)