[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 171 **G** **Your Roll No.....**

Unique Paper Code : 235151

Name of the Paper : Mathematics : Calculus

Name of the Course : **B.A. (Prog.) Mathematics**

Semester : I

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** questions are compulsory.
3. Attempt any **two** parts from each question.

1. (a) Prove that $\lim_{x\to 0} \frac{x-|x|}{x}$ does not exist. (6)

(b) Discuss the derivability of the function

$$f(x)=\begin{cases} x & , \text{ for } x<1 \\ 2-x & , \text{ for } 1\le x\ 2 \\ -2+3x-x^2 & , \text{ for } x>2 \end{cases}$$

at $x=1,2.$ (6)

P.T.O.

(c) Show that the function

$$f(x)=\begin{cases}\dfrac{e^{1/x}-e^{-1/x}}{e^{1/x}+e^{-1/x}} & , x\neq 0\\ 0 & , x=0\end{cases}$$

is discontinuous at $x = 0$. (6)

2. (a) If $x = a(\cos\theta + \theta\sin\theta)$, $y = a(\sin\theta - \theta\cos\theta)$

find $\dfrac{d^2y}{dx^2}$. (6½)

(b) If $y = e^{m.\sin^{-1}x}$, show that

$$(1+x^2)y_{n+2}-(2n+1)xy_{n+1}-(n^2+m^2)y_n=0.$$

(c) If $z=\tan^{-1}\left(\dfrac{y}{x}\right)$, Verify that $\dfrac{\partial^2 z}{\partial x^2}+\dfrac{\partial^2 z}{\partial y^2}=0$. (6½)

3. (a) Find the equations of the tangent and normal to the curve

$$y(x-2)(x-3)-x+7=0$$

at the point where it meets the x axis. (6)

(b) The tangent at any point of the curve $x = a\cos^3\theta$, $y = b\sin^3\theta$ meets the co-ordinate axes at points A and B respectively. Show that the locus of the point with (OA,OB) as co-ordinates is

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1. \qquad (6)$$

(c) Find the points on the parabola $y^2 = 8x$ at which the radius of curvature is $7\frac{13}{16}$. (6)

4. (a) Find all the asymptotes of the curve

$$x^3 + 2x^2y - xy^2 - 2y^3 + 4y^2 + 2xy + y + 1 = 0. \qquad (6½)$$

(b) Determine the position and nature of the double points on the curve

$$x^3 - 7x^2 - y^2 + 15x + 4y - 13 = 0. \qquad (6½)$$

(c) Trace the curve $y^2 = (x-2)(x-5)^2$. (6½)

5. (a) State Lagrange's mean value theorem. Verify it for the function

$$f(x) = (x-1)(x-2)(x-3)$$

in the interval [1,4]. (6)

(b) Find Maclaurin's power series expansion of the function $f(x) = e^x$. (6)

(c) Verify Cauchy's mean value theorem for the functions x^2 and x^3 in the interval [1,2]. (6)

6. (a) Use Lagrange's mean value theorem to the function $\tan^{-1} x$ to show that

$$\frac{u-v}{1+v^2} < \tan^{-1} v - \tan^{-1} u < \frac{v-u}{1+u^2}, \text{ if } 0 < u < v. \qquad (6½)$$

(b) Evaluate $\lim_{x\to 0} \dfrac{1-\cos\left(x^2\right)}{x^2 \sin\left(x^2\right)}$. (6½)

(c) Find the largest and the smallest values of the polynomial $x^3 - 18x^2 + 96x$ in the interval [0,9]. (6½)