[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 172 **G** **Your Roll No................**

Unique Paper Code : 235151

Name of the Paper : Mathematics : Calculus

Name of the Course : **B.A. (Prog.) Mathematics**

Semester : I

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** question are compulsory.
3. Attempt any **two** parts from each question.

1. (a) Show that $\lim_{x \to 1} \frac{|x-1|}{x-1}$ does not exist. (6)

(b) Examine the continuity of the function

$$f(x) = \begin{cases} (1+2x)^{\frac{1}{x}} & , \quad x \neq 0 \\ e^2 & , \quad x = 0 \end{cases}$$

at $x = 0$. (6)

(c) Show that the function f defined as

P.T.O.

$$f(x) = \begin{cases} \dfrac{x-1}{2x^2-7x+5} & , \quad x \neq 1 \\ -\dfrac{1}{3} & , \quad x = 1 \end{cases}$$

is differentiate at $x = 1$ and hence find $f'(1)$. (6)

2. (a) Find the n^{th} derivative of $e^x \sin^4 x$. (6½)

(b) If $y = \tan^{-1} x$, show that

$$\left(1+x^2\right) y_{n+2} + (2n+1) x y_{n+1} + n(n+1) y_n = 0$$ (6½)

(c) If $u = \log(x^2 + y^2 + z^2)$, prove that

$$x \frac{\partial^2 u}{\partial y \, \partial z} = y \frac{\partial^2 u}{\partial z \, \partial x} = z \frac{\partial^2 u}{\partial x \, \partial y}.$$ (6½)

3. (a) Show that the length of the portion of the tangent to the curve $x = a \cos^3 \theta$, $y = a \sin^3 \theta$ intercepted between the co-ordinate axes is constant. (6)

(b) Show that the tangent at any point of the curve $x = a(t + \sin t.\cos t)$, $y = a(1 + \sin t)^2$ makes an angle $\frac{1}{4}(\pi + 2t)$ with x axis. (6)

(c) The tangents at two points P, Q on the cycloid $x = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$ are at right angles, show that if ρ_1, ρ_2 be the radii of curvature at these points, then

$$\rho_1^2 + \rho_2^2 = 16a^2. \qquad (6)$$

4. (a) Find all the asymptotes of the curve

$$x^3 + 3x^2y - 4y^3 - x + y + 3 = 0. \qquad (6½)$$

(b) Determine the position and nature of the double points on the curve

$$x^3 - 2x^2 - y^2 + x + 4y - 4 = 0. \qquad (6½)$$

(c) Trace the curve $y^2x = a^2(a - x)$. (6½)

5. (a) By considering the function $f(x) = (x - 2)\log x$ in $[1,2]$, show that the equation $x\log x = 2 - x$ is satisfied by at least one value of x lying between 1 *and* 2. (6)

(b) Find Maclaurin's power series expansion of the function $f(x) = \sin 5x$. (6)

(c) Find 'c' using Lagrange's mean value theorem

if $f(x) = x(x-1)(x-2);\ a = 0,\ b = 1/2\ \textit{where}\ a < c < b.$ (6)

6. (a) Show that the function f, defined by $f(x) = x^5 - 5x^4 + 5x^3 - 1$ for all $x \in \Re$ has a maximum value when $x = 1$, minimum value when $x = 3$ and neither when $x = 0$. (6½)

(b) Evaluate $\lim_{x\to 0}\left[\frac{1}{x} - \frac{\log(1+x)}{x^2}\right]$. (6½)

(c) Use Lagrange's mean value theorem to the function $log(1 + x)$ to show that

$$0 < \frac{1}{\log(1+x)} - \frac{1}{x} < 1 \text{ for all } x > 0.$$ (6½)