[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 5335 **D** **Your Roll No...............**

Unique Paper Code : 235251

Name of the Course : **B.A. (Prog.)**

Name of the Paper : Mathematics (Algebra)

Semester : II

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **two** parts from each question.

1. (a) Prove that the set of all matrices of the form $\begin{pmatrix} x & 0 \\ y & z \end{pmatrix}$, where x, y, z ∈ C is a vector space over C with respect to matrix addition and multiplication of a matrix by a scalar. C denotes the set of Complex numbers. (6)

   (b) Define linearly independent set of vectors and show that the following set of vectors (1,1,0,0), (0,1,–1,0) and (0,0,0,3) in R^4 are linearly independent over R. (6)

   (c) Define subspace of a vector space. Show that the set

   $\omega = \{(a, b, c): a + b + c = 0,\ a, b, c \in R\}$ is a subspace of R^3. (6)

2. (a) Obtain the rank of the matrix

$$\begin{pmatrix} 1 & 2 & 4 \\ -1 & -2 & -4 \\ 2 & 4 & 8 \\ -3 & -6 & -9 \end{pmatrix}$$ (6½)

(b) For what values of λ, does the following system of equations have a solution :

$x + y + z = 1$

$x + 2y + 4z = \lambda$

$x + 4y + 10z = \lambda^2$

also find the solution. (6½)

(c) Verify Cayley Hamilton Theorem for the matrix $A = \begin{pmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{pmatrix}$.

And use it to find the inverse of A i.e. A^{-1}. (6½)

3. (a) Find the modulus and argument of the complex number

$$\frac{(1+\cos\theta+i\sin\theta)^5}{(\cos\theta+i\sin\theta)^3}$$ (6)

(b) Prove that

$32\sin 4\theta\cos 2\theta = \cos 6\theta - 2\cos 4\theta - \cos 2\theta + 2.$ (6)

(c) Sum upto n terms

$\cos\alpha + \cos(\alpha+\beta) + \cos(\alpha+2\beta) + \ldots\ldots\ldots$ to n terms

provided $\beta \neq 2k\pi$, what is the sum if $\beta = 2k\pi$? (6)

4. (a) Solve the equation

$$3x^4 - 40x^3 + 130x^2 - 120x + 27 = 0$$

Given that the product of two of its roots is equal to the product of the other two. (6½)

(b) Solve the equation :

$$x^3 - 13x^2 + 15x + 189 = 0$$

being given that one of the roots exceeds another by 2. (6½)

(c) If α, β, γ be the roots (all non zero) of the equation $x^3 - px^2 + qx - r = 0$. Find the values of

(i) $\sum\left(\alpha/\beta\right)$

(ii) $\sum\alpha^2$ (6½)

5. (a) Show that the set {1, 5, 7, 11} is a group with respect to multiplication modulo 12. (6)

(b) If G is a group if a, b ∈ G, then prove that

$$(a \cdot b) = (b \cdot a) \Rightarrow (a \cdot b)^n = a^n \cdot b^n,$$

n being any positive integer. (6)

(c) Consider the following permutations in S_4

$$f = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 1 & 3 \end{pmatrix} \text{ and } g = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 2 & 4 & 1 \end{pmatrix}$$

Compute the following : (i) fg (ii) f^2g. (6)

6. (a) Let G = {(a,b): a ≠ 0 and a, b ∈ R} and '*' be a binary operation defined by (a,b) * (c,d) = (ac, bc + d). Show that (G, *) is a non-abelian group. (6½)

(b) Prove that set Q of all rational numbers is a commutative ring with unity, the addition and multiplication of rational numbers being two ring compositions. (6½)

(c) Prove that rigid motions of a square yield the group S_4. (6½)