[This question paper contains 2 printed pages.]

Sr. No. of Question Paper : 5336 **D** **Your Roll No...............**

Unique Paper Code : 235251

Name of the Course : **B.A. (Prog.)**

Name of the Paper : Mathematics (Algebra)

Semester : II

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **two** parts from each question.

1. (a) Let S be the set of all matrices of the form

$$\begin{pmatrix} a & b \\ -b & a \end{pmatrix}.$$

where a, b are any complex numbers. Then show that S is a vector space over C with respect to matrix addition and multiplication by a scalar. (6)

(b) If x, y, z are linearly independent elements of a vector space over R, the field of real numbers, show that $x + 3y - 2z$, $2x + y - z$ and $3x + y + z$ are also linearly independent. (6)

(c) Define bases of a vector space and show that the vectors (1,2,1), (2,1,0) and (1,–1,2) form a basis of $R^3(R)$. (6)

2. (a) Reduce the following matrix in triangular form and hence find its rank.

$$\begin{pmatrix} 1 & 1 & 2 & 3 \\ 1 & 3 & 0 & 3 \\ 1 & -2 & -3 & -3 \\ 1 & 1 & 2 & 3 \end{pmatrix}$$ (6½)

(b) Is the following system of equations consistent ?

$5x + 3y + 14z = 4$
$y + 2z = 1$
$x - y + 2z = 0$
$2x + y + 6z = 0,$

if yes find the solution. (6½)

(c) Find the characteristic equation and its roots of the matrix :

$$A = \begin{pmatrix} 3 & 4 & 5 \\ 0 & 9 & 0 \\ 0 & 7 & 2 \end{pmatrix}$$ (6½)

3. (a) Solve the equation using De Moivre's Theorem

$$Z^{10} - Z^5 + 1 = 0.$$ (6)

(b) Prove the following identity

$64 \sin^3\theta \cos^4\theta = -\sin 7\theta - \sin 5\theta + 3\sin 3\theta + 3\sin\theta.$ (6)

(c) Sum of n terms :

$\sin\theta + x\sin 2\theta + x^2\sin 3\theta + \dots\dots\dots\dots + n$ terms. (6)

4. (a) Solve the equation :

$3x^3 + 11x^2 + 12x + 4 = 0$, being given that the roots are in H.P. (6½)

(b) Solve the equation :

$$x^4 + 15x^3 + 70x^2 + 120x + 64 = 0$$

the roots being in GP. (6½)

(c) If α, β, γ be the roots (all non zero) of the equation $x^3 + px^2 + qx + r = 0$. Find the values of

(i) $\sum \alpha^2\beta^2$

(ii) $\sum \alpha^2\beta$ (6½)

5. (a) Show that the set S = {0, 1, 2, 3, 4} is an abelian group with respect to addition modulo 5. (6)

(b) A non-empty subset H of a group G is a subgroup of G if and only if

(i) $a \in H, b \in H \Rightarrow ab \in H$

(ii) $a \in H \Rightarrow a^{-1} \in H$ where a^{-1} is the inverse of a in G. (6)

(c) Find the powers of f = (1 2 3 4 5) that is f^2, f^3, f^4, f^5 where $f \in S_5$. (6)

6. (a) Prove that the set I of all integers with the binary operation '*' defined by $a * b = a + b + 1$ for all $a, b \in I$ is an abelian group. (6½)

(b) Prove that set 2I of all even integers is a commutative ring without unity, the addition and multiplication of integers being two ring compositions. (6½)

(c) Prove that rigid motions of a square yield the group S_4. (6½)