This question paper contains 4 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **5328**

Unique Paper Code : **237251** **D**

Name of the Paper : **Statistical Methods-I**

Name of the Course : **B.A. (Prog.) Statistics**

Semester : **II**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *six* questions in all.

Question No. **1** is compulsory.

Attempt **5** more questions.

1. (*a*) Fill in the blanks :

(*i*) A continuous distribution for which mean = variance is ....

(*ii*) If $X \sim N(4, 25)$, mode of the distribution is ....

(*iii*) Variance of binomial distribution with parameters n and p is ....

(*iv*) Coefficient of variation for Poisson distribution with mean 4 is ....

(*v*) If $V(X) = 2$, then $V(5 - X)$ is ....

P.T.O.

(*b*) If X and Y are independent standard normal variates then find the distribution of X − 2Y.

(*c*) The m.g.f. of a random variable X is $M_x(t) = \exp[3(e^t - 1)]$, using the uniqueness property of m.g.f.'s identify the distribution and its parameters.

(*d*) Let X denote the number of successes preceeding the *r*th failure. Find E(X).

(*e*) A Poisson variate X is such that P(Y = 2) = P(Y = 3), find its mean and coefficient of skewness. 1×5,2,2,2,2,2

2. (*a*) For the given probability law :

$$dF = kx^2 e^{-x} dx; \quad 0 < x < \infty$$

Find mean, variance, β_1 and β_2 for the distribution.

(*b*) A random variable X assumes the values −3, −2, −1, 0, 1, 2, 3 such that P[X = −3] = P[X = −2] = P[X = −1], P[X = 1] = P[X = 2] = P[X = 3], and P[X = 0] = P[X > 0] = P[X < 0]. Obtain the p.m.f. of X and its distribution function, and further find the p.m.f. of $Y = 2X^2 + 3X + 4$. 6,6

3. (*a*) State and prove De-Moivre's Laplace theorem.

(*b*) Examine whether the weak law of large numbers holds for the sequence $\{X_m\}$ of independently and identically distributed random variable where :

$$P(X_i = (-1)^{k-1} \cdot k) = \frac{6}{k^2\pi^2};\ k = 1, 2, 3, \ldots\ldots;\ i = 1, 2, 3, \ldots\ldots$$

(*ii*) The r.v.'s $X_1, X_2, \ldots\ldots X_n$ have equal expectations and finite variation. Is the weak law of large numbers applicable to this sequence if all the co-variances σ_{ij} are negative ? 6,6

4. (*a*) The joint probability mass function of two discrete random variables X and Y is given by $p(x, y) = c(2x + y)$, where x and y can assume all integers such that $0 \leq x \leq 2$, $0 \leq y \leq 3$, and $p(x, y) = 0$ otherwise. Find (*i*) value of constant c; (*ii*) $P(X = 2, Y = 1)$; (*iii*) $P(X \geq 1, Y \leq 2)$, (*iv*) $P(X = x|Y = 2)$.

(*b*) Obtain Poisson distribution as a limiting case of negative binomial distribution. 6,6

5. (*a*) Define hypergeometric distribution and compute its mean and variance.

(*b*) If $X \sim N(\mu, \sigma^2)$, obtain its mean deviation about mean. 6,6

6. (*a*) If X is a binomial variate with parameters n and p, find the m.g.f. standard binomial variate and obtain its limiting form as $n \to \infty$. Also interpret the result.

(*b*) If $X \sim$ Poisson (λ), find mode of the distribution. 6,6

7. (*a*) Show that for the symmetrical distribution :

$$f(x) = \frac{2a}{\pi}\left(\frac{1}{a^2 + x^2}\right); \qquad -a \leq x \leq a,$$

$$\mu_2 = \frac{a^2(4 - \pi)}{\pi} \text{ and } \mu_4 = a^4\left(1 - \frac{8}{3\pi}\right).$$

(*b*) Compute harmonic mean for beta variate of first kind. 6,6

8. (*a*) Let random variable X follow $N(\mu, \sigma^2)$. Find its m.g.f. and hence deduce that all odd order central moments vanish and $\mu_{2n} = 1.3.5. \ldots\ldots (2n - 1)\, \sigma^{2n}$

(*b*) If X has an geometric distribution with parameter p, then for every constant $a \geq 0$ $P(Y = t | X \geq a) = P(X \leq t)$ for all t, where $Y = X - a$. 6,6