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Your Roll No..........................

5436

B.A. Programme/II J

(R)

MATHEMATICS—Paper II

(Geometry, Differential Equations and Algebra)

(NC—Admissions of 2004 and onwards)

Time : 3 *Hours* *Maximum Marks* : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

Note :— The maximum marks printed on the question paper are applicable for the students of the regular colleges (Cat. 'A'). These marks will, however, be scaled up proportionately in respect of the students of NCWEB at the time of posting of awards for compilation of result.

All questions are compulsory.

Attempt any *two* parts from each question.

1. (a) Describe the graph of the equation :

$$x^2 - y^2 - 4x + 8y - 21 = 0.$$

Find the centre and the foci. 6

(b) Identify and sketch the curve :

$$x^2 + 9y^2 + 2x - 18y + 1 = 0.$$ 6

(c) Find an equation for a hyperbola whose vertices are $(\pm 1, 0)$ and asymptotes are $y = \pm 2x$. 6

2. (a) Find the equation of the sphere that has $(1, -2, 4)$ and $(3, 4, -12)$ as end points of a diameter. Also find its centre and radius. 6½

(b) (i) Find the angle between vectors

$$\vec{a} = 4\hat{i} - 2\hat{j} + 4\hat{k} \text{ and } \vec{b} = 3\hat{i} - 6\hat{j} - 2\hat{k}.$$

(ii) Find the parametric equation of the line that is perpendicular to lines :

$L_1 : x = 4t,\ y = 1 - 2t,\ z = 2 + 2t$ and

$L_2 : x = 1 + t,\ y = 1 - t,\ z = -1 + 4t$

and passes through their points of intersection. 3+3½

(c) (i) Find the equation of the plane through the point (–1, 2, –5) and perpendicular to planes $2x - y + z = 1$ and $x + y - 2z = 3$.

(ii) Show that the line $x = -1 + t,\ y = 3 + 2t,\ z = -t$ and the plane $2x - 2y - 2z + 3 = 0$ are parallel and find distance between them. 3+3½

3. (a) (i) Solve the equation :

$$(5xy + 4y^2 + 1)\,dx + (x^2 + 2xy)\,dy = 0.$$

(ii) Solve the equation :

$$x_1 + 2x - 3y = t$$

$$y_1 - 3x + 2y = e^{2t}$$

where

$$x_1 = \frac{dx}{dt}, \quad y_1 = \frac{dy}{dt}.$$

3½+4

(b) (i) Solve the equation :

$$y'' + 4y = \sin^2 2x$$

by the method of variation of parameters.

(ii) Solve :

$$yz(1 + x)\, dx + zxy(1 + y)\, dy + xy(1 + z)\, dz = 0.$$

4+3½

(c) (i) Assume that the population of a certain city increases at a rate proportional to the number of inhabitants at any time. If the population doubles in 40 years, in how many years will it triple itself ?

(ii) Solve the equation :

$$(D^2 - 4D + 4)\,y = x^2 + \sin 2x + e^x.$$ 3½+4

4. (a) Find the complete integral of the equation :

$$pxy + pq + qy - yz = 0.$$ 5

(b) (i) Find the general integral of the differential equation :

$$\frac{y-z}{yz}\,p + \frac{z-x}{zx}\,q = \frac{x-y}{xy}.$$

(ii) Find whether the equation :

$$x^2 r + \frac{5}{2}xys + y^2 t + xp + yq = 0$$

is elliptic, parabolic or hyperbolic. 4+1

(c) (i) Find the complete integral of the equation :

$$p^2 z^2 + q^2 = 1.$$

(ii) Eliminate the arbitrary function f from the equation :

$$z = x + y + f(xy).$$ 3+2

5. (a) (i) Express $\sigma = (1\ 2\ 3)(1\ 5\ 7)(2\ 6\ 7)$ as a product of disjoint cycles in S_7. Find the inverse and order of σ.

(ii) Let $GL_2(R)$ be the set of all 2×2 invertible metrices with entries in R. Show that it forms a group under matrix multiplication. Is it an abelian group ? Justify your answer. 3+3½

(b) (i) Let H be a non-empty subset of a group G. Prove that H is a subgroup of G if and only if :

$ab^{-1} \in H$ for all $a, b \in H$.

(ii) If a ring R is such that $x^2 = x$ for all $x \in R$, then prove that :

(1) $2x = 0$

(2) $x + y = 0$ implies $x = y$

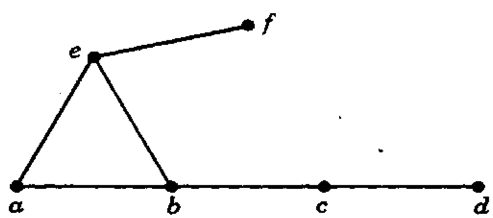
(3) R is commutative. 3+3½

(c) (i) Show that every proper subgroup of order 21 is cyclic.

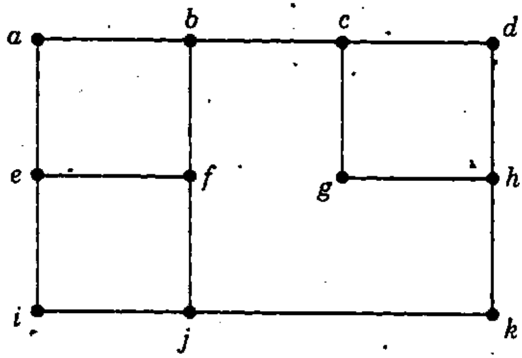
(ii) If H and K are subgroups of a group G, show that $H \cap K$ is also a subgroup of G. 3½+3

6. (a) (i) Define a Latin square. Give an example of a Latin square of order 6.

(ii) Is the following graph an interval graph ? Justify. 2+4



(*b*) Find a minimal edge cover for the following graph. Give a detailed logical analysis. 6



(*c*) (*i*) Three pitchers of sizes 10*l*, 4*l* and 7*l* are given. If initially 10*l* pitcher is full and the other two empty, find a minimal sequence of pouring so as to have exactly 2*l* in either the 7*l* or the 4*l* pitcher.

(*ii*) If C_{23} is the cost of travelling from 2 to 3, should it be used in the following cost matrix for travelling salesperson problem ? Justify. 3+3

	To	1	2	3	4
	1	—	7	1	3
From	2	7	—	4	5
	3	5	4	—	4
	4	1	3	6	—