This question paper contains 8+1 printed pages]

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B.A. Programme/II   J

(A)

MATHEMATICS—Paper II

(Geometry, Differential Equations and Algebra)

(NC—Admissions of 2004 and onwards)

Time : 3 *Hours*   *Maximum Marks* : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

Note :—   .The maximum marks printed on the question paper
are applicable for the students of the regular colleges
(Cat. 'A'). These marks will, however, be scaled
up proportionately in respect of the students of
NCWEB at the time of posting of awards for
compilation of result.

All questions are compulsory.

Attempt any *two* parts from each question.

1. (a) Sketch the hyperbola :

$$16x^2 - y^2 - 32x - 6y = 57$$

and label the vertices, foci and asymptotes.   6

(b) Identify and sketch the curve :

$$153x^2 - 192xy + 97y^2 - 30x - 40y - 200 = 0.$$ 6

(c) Find an equation of a parabola whose vertex is
(5, –3), axis parallel to the y-axis and the parabola
passes through (9, 5).   6

2. (a) Find the equation of the sphere passing through
three points (3, 0, 2), (–1, 1, 1), (2, –5, 4) and having
its centre on the plane :

$$2x + 3y + 4z = 6.$$ 6½

(b) (i) Prove that :

$$\vec{u} \times (\vec{v} \times \vec{w}) = (\vec{u} \cdot \vec{w})\, \vec{v} - (\vec{u} \cdot \vec{v})\, \vec{w} \cdot$$

(ii) Find the angle between the vectors
$2\hat{i} - 3\hat{j} + 6\hat{k}$ and $\hat{i} + 2\hat{j} + 2\hat{k}$.   3½+3

(c) (i) Find the parametric equation of the line passing through the point (–1, 2, 4) and parallel to vector $3\hat{i} - 4\hat{j} + \hat{k}$.

(ii) Find the equation of plane through points (1, 2, –1), (2, 3, 1) and (3, –1, 2).   3+3½

3. (a) (i) Solve the equation :

$$y = 2px - xp^2$$

where :

$$p = \frac{dy}{dx}.$$

(ii) Solve the equation :

$$y'' + 4y = 4\tan 2x$$

by the method of variation of parameters.

3½+4

(b) (i) Solve the equation :

$$(y^2 + z^2 - x^3)\, dx - 2xy\, dy - 2xz\, dz = 0.$$

(ii) Find the orthogonal trajectories of the family of curves :

$$x^2 + y^2 = cx^3.$$ 3½+4

(c) (i) State a necessary and sufficient condition for the solutions $y_1(x)$ and $y_2(x)$ of the equation :

$$a_0(x) \, y'' + a_1(x) \, y' + a_2(x) \, y = 0$$

to be linearly independent. Show that e^{2x} and e^{3x} are linearly independent solutions of the equation :

$$y'' - 5y' + 6y = 0 \text{ on } -\infty < x < \infty.$$

What is the general solution ? Find the solution $y(x)$ that satisfies the conditions :

$$y(0) = 2, \ y'(0) = 3.$$

(ii) Solve the equation :

$$(D^2 + 4) \, y = \sin 3x + e^x + x^2.$$ 4+3½

4. (a) Find the complete integral of the equation :

$$2(z + xp + yq) = yp^2.$$ 5

(b) (i) Find the general integral of the differential

equation :

$$px (z - 2y^2) = (z - qy) (z - y^2 - 2x^3).$$

(ii) Find whether the equation :

$$x^2r - y^2t - px - qy = x^2$$

is hyperbolic, parabolic or elliptic.   4+1

(c) (i) Find the complete integral of the

equation :

$$pqz = p^2 (xq + p^2) + q^2 (yp + q^2).$$

(ii) Eliminate the constants a and b from the

equation :

$$ax^2 + by^2 + z^2 = 1.$$ 3+2

5. (a) (i) Write :

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 3 & 9 & 5 & 4 & 7 & 8 & 1 & 6 & 10 & 2 \end{pmatrix}$$

as

(i) a product of disjoint cycles

(ii) product of transpositions.

Find the inverse and order of σ.

(ii) Find the group of symmetries of a rectangle. 3+3½

(b) (i) Show that the set R = {2m : m is an integer} is a commutative ring for ordinary addition and multiplication.

(ii) Prove that the order of every element of a finite group divides the order of the group. 3+3½

(c) (i) Prove that $H_m = \{nm/n \in Z\}$ is a subgroup of (Z, +), where Z is the set of integers and $m \in Z$.

(*ii*)   Let $G = R - \{1\}$, the set of all reals excluding 1. Define a binary operation * on G by :
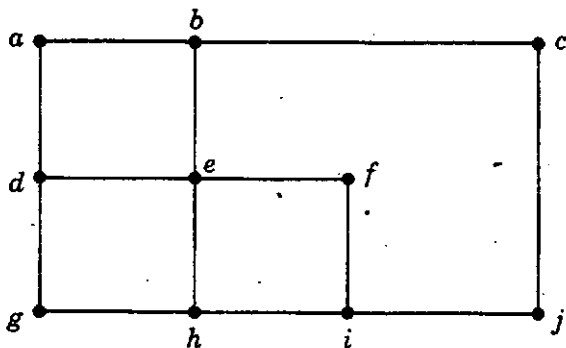
$$a * b = a + b - ab.$$

Show that (G, *) is a group.   3+3½

6.   (*a*)   (*i*)   Can the given Latin square be obtained from the multiplication table from a group of order 6 with identity A ? Give reasons to support your answer.
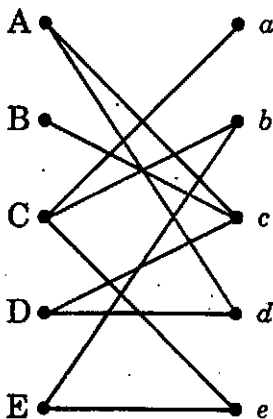
A	B	C	D	E	F
B	A	F	E	C	D
C	F	B	A	D	E
D	C	E	B	F	A
E	D	A	F	B	C
F	E	D	C	A	B

(ii) Find a maximum independent set for the
      following graph. What is the minimum
      number of independent sets needed to cover
      all the vertices ?   3+3

(b) (i) Find a matching or explain why none exists
          for the following graphs :

(*ii*) Three pitchers of sizes 12*l*, 8*l* and 5*l* are given. If initially the 12*l* pitcher is full and the other two empty, find a minimal sequence of pouring so as to have exactly 1*l* in either the 8*l* or the 5*l* pitcher.   3+3

(*c*) Solve the travelling salesperson problem for the given cost matrix :   6

	To	1	2	3	4
	1	—	7	2	6
From	2	1	—	8	9
	3	2	1	—	3
	4	4	2	6	—