This question paper contains 8 printed pages.]

4582 *Your Roll No. .....*

B.A. Prog. / II **AS**

(L)

OPERATIONAL RESEARCH

Paper II – Operational Research-I

(Admissions of 2004 and onwards)

Time : 3 Hours *Maximum Marks : 75*

(Write your Roll No. on the top immediately on receipt of this question paper.)

*Answer **six** questions in all, selecting at least **one** question from each section. All questions carry equal marks.*

Section A : Linear Programming

1. (a) Formulate a mathematical model for a general linear programming problem in n-variables and define the following terms (i) a feasible solution (ii) a basic feasible solution (iii) an optimal solution.

[P.T.O.

(b) Solve the following linear programming problem using graphical method :

Maximize $Z = x_1 + 2x_2$

Subject to $-x_1 + 3x_2 \leq 10$

$x_1 + x_2 \leq 6$

$x_1 - x_2 \leq 2$

$x_1 + 3x_2 \geq 6$

$x_1, x_2 \geq 0$

2. (a) In solving a linear programming problem by simplex method, explain fully, discussing all cases, how you will move from a given basic feasible solution to a better basic feasible solution.

(b) Solve the following linear programming problem by simplex method, using Charne's M-technique to handle artificial variables.

Minimize $Z = 2x_1 + x_2$

subject to $3x_1 + x_2 = 3$

$4x_1 + 3x_2 \geq 6$

$x_1 + 2x_2 \leq 3$

$x_1, x_2 \geq 0.$

3. (a) Write the dual of the following linear programming problem in such a way that the dual

variables as well as the requirement vector of the dual are all non-negative :

Minimize $Y = 5y_1 - 7y_2 + 9y_3$

subject to $y_1 + 2y_2 + 3y_3 \geq 100$

$10y_1 + 11y_2 - 12y_3 \leq 50$

$27y_1 + 5y_2 = 75$

$y_1, y_2 \geq 0 \quad y_3 \geq 5$

(b) List the various methods that can be used for obtaining the Initial basic feasible solution of a transportation problem and explain any one of the methods in detail.

4. (a) Five men are available to do five different jobs. From past records, the time (in hrs) that each man takes to perform each of these jobs is known and given in the following table :

		Job				
		I	II	III	IV	V
Man	A	11	17	18	16	20
	B	9	7	12	6	15
	C	13	16	15	12	16
	D	21	24	17	28	26
	E	14	10	12	11	15

How should the jobs be allocated, one to a man, so as to minimize the total man hours ?

(b) State the general rule of dominance for two-person zero-sum games and use it to solve the game whose pay-off matrix is

$$\begin{vmatrix} 10 & 0 & 6 \\ 0 & 10 & 7 \end{vmatrix}.$$

Section B : Inventory Management

5. (a) An item is produced at the rate of 128 units per day. The annual demand is 6400 units. The set up cost for each production run is Rs. 24/- and the cost of carrying inventory is Rs. 3/- per unit per day. There are 250 working days for production each year. Find the optimum lot size and minimum total cost involved.

(b) In a certain manufacturing situation, the production is instantaneous and demand per day is R. Show that the optimum order quantity q per run that minimizes the total cost is given by

$$q = \sqrt{\frac{2RC_s(C_1 + C_2)}{C_1C_2}}$$

where

(i) C_1 is the cost of holding one unit of inventory per day

(ii) C_2 is the unit shortage cost per day

(iii) C_s is the set up cost per production run.

6. (a) What are the various costs that are associated with an inventory process ? Discuss the important components that constitute the stock holding costs.

(b) Classify the following inventory items into different classes using ABC analysis with its graphical representation.

Item No.	Annual Consumption (Unit)	Unit Cost (Rs.)
1	300	10
2	2800	15
3	30	10
4	1100	5
5	40	5
6	220	100
7	1500	5
8	800	5
9	600	15
10	80	10

7. (a) What do you mean by buffer stock ? Explain how reserve stock level is found for a normally distributed demand during the lead time.

(b) A particular item has a demand of 900 units/year. The cost of one procurement is Rs. 100/- and the holding cost is Rs. 2.40/ per unit per year. Given that the replenishment is instantaneous and cost of shortage is Rs. 5 / unit/ year, determine the economic lot size and the total cost per year if the cost of one unit is Re. 1/-.

Section C : Queueing Theory

8. (a) For a queueing model $M|M|I|\infty|FIFO$, derive expressions for :

(i) Average number of units in the system

(ii) Average length of a non-empty queue

(b) In a certain coffee shop it was observed that there is only one waiter who takes exactly 4 minutes to serve a cup of coffee once the order has been placed with him. If the students arrive in the coffee shop at an average rate of 10 per hour, how much time must a student expect to spend in the shop waiting to get his cup of coffee.

9. (a) Derive steady state solution for the single server finite capacity queueing model $M|M|I|K|FIFO$.

(b) There are two clerks in a university office to receive dues from the students. If the service time for each student is exponential with mean 4 minutes and the students arrive in a Poisson fashion at the counter at the rate of 10 per hour :

(i) What is the probability of having to wait for service?

(ii) What is the expected percentage of idle time for each clerk?

10. (a) Explain the generalized birth death queueing model along with its special cases.

(b) Assume that the goods trains are coming in a yard at the rate of 30 trains per day and suppose that the interarrival times of these trains follow an exponential distribution. The service time (the

time taken to bump a train) distribution is also exponential with an average of 36 minutes, calculate the following :

(i) the probability that yard is empty

(ii) expected waiting time in the queue.