This question paper contains 7 printed pages.]

Your Roll No. ....

883

B.A. Programme / II A

(L)

MATHEMATICS – Paper II

(Geometry, Differential Equations and Algebra)

(Admissions of 2004 and onwards)

Time : 3 Hours **Maximum Marks : 75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Note : The maximum marks printed on the question paper are applicable for the students of the regular colleges (Cat. 'A'). These marks will, however, be scaled up proportionately in respect of the students of NCWEB at the time of posting of awards for compilation of result.

All questions are compulsory.

Attempt any **two** parts from each question.

1. (a) (i) Sketch the graph of the equation $x^2 - 4x + 2y = 1$ and label the focus, vertex and directrix. **4**

(ii) Find an equation for a hyperbola with foci $(0, \pm 5)$ and asymptotes $y = \pm 2x$. **2½**

(b) Identify and sketch the curve

$$x^2 + 3y^2 + 4x + 6y + 1 = 0$$

Also label the foci, the vertices and the ends of minor axis. **6½**

(c) Consider the equation

$$x^2 + 4xy - 2y^2 - 6 = 0$$

Rotate the co-ordinate axes to remove the xy-term and then identify the type of the conic represented by the equation. **6½**

2. (a) Find the equation of the sphere which passes through the points (3, 0, 2), (–1, 1, 1), (2, –5, 4) and whose center lies on the plane $2x + 3y + 4z = 6$. **6**

(b) (i) Determine whether to vectors

$\vec{u} = (5, -2, 1)$, $\vec{v} = (4, -1, 1)$,

$\vec{w} = (1, -1, 0)$ lie in the same plane.

(ii) Using vectors, find the area of the triangle with vertices P_1 (3, –1, 2), P_2 (1, –1, –3) and P_3 (4, –3, 1). **3 + 3**

(c) (i) Show that the lines

$L_1 : x = -2 + t, y = 3 + 2t, z = 4 - t; -\infty < t < \infty$

$L_2 : x = 3 - t, \quad y = 4 - 2t, z = t \quad ; -\infty < t < \infty$

are parallel and find an equation of the plane they determine.

(ii) Find the coordinates of the point where the line

$\vec{r} = (1 + t)\,\hat{i} + (-1 + 3t)\,\hat{j} + (2 + 4t)\,\hat{k}; -\infty < t < \infty$

intersects the plane $x - y + 4z = 7$

3½ + 2½

3. (a) (i) Test the differential equation

$(a^2 - 2xy - y^2)\,dx - (x + y)^2\,dy = 0$

for exactness & hence solve it.

(ii) Solve

$$x^2 \frac{d^2y}{dx^2} - x\frac{dy}{dx} + 2y = x \log x$$ **3 + 3½**

(b) (i) Solve

$$p^2y + p(x - y) - x = 0$$

(ii) Show that e^{-x}, e^{3x} and e^{4x} are linearly independent solutions of

$$\frac{d^3y}{dx^3} - 6\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 12y = 0$$

on the interval $-\infty < x < \infty$ and write the general solution. 3 + 3½

(c) (i) Solve

$\frac{d^2y}{dx^2} + y = \sec^3 x$, using method of variation of parameters.

(ii) Solve the system of equations

$$\frac{dx}{dt} - 7x + y = 0$$

$$\frac{dy}{dt} - 2x - 5y = 0$$

3 + 3½

4. (a) (i) Here $p = \frac{\partial z}{\partial x}$, $q = \frac{\partial z}{\partial y}$, $r = \frac{\partial^2 z}{\partial x^2}$, $r = \frac{\partial^2 z}{\partial x \partial y}$ and $t = \frac{\partial^2 z}{\partial y^2}$.

Form a partial differential equation by eliminating the arbitrary function f possessing partial derivatives of the first order from the equation $z = f(x - y)$

Examine whether the partial differential equation so obtained is linear or non-linear.

(ii) Find a complete integral of the linear partial differential equation :

$$p^2z^2 + q^2 = 1 \qquad 4 + 2$$

(b) (i) Find the general integral of the linear partial differential equation

$$\cos(x + y)p + \sin(x + y)q = z + \frac{1}{z}.$$

(ii) Classify the following second order partial differential equation into elliptic, parabolic or hyperbolic equation : **5 + 1**

$$x^2r - y^2t + px - qy = x^2.$$

(c) Find a complete integral of the equation

$$px^5 - 4q^3x^2 + 6x^2z - 2 = 0. \qquad 6$$

5. (a) (i) Let G be a group in which $x^{-1} = x$ $\forall x \in G$. Prove that G is abelian. **3½**

(ii) Let 5 = {1, 3, 7, 9}. Show that $(S, \odot_{10})$ is an abelian group where $\odot_{10}$ is multiplication modulo 10. **3**

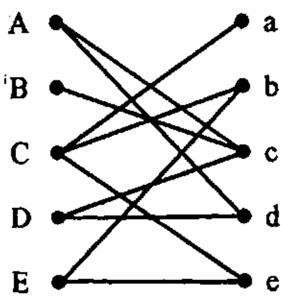
(b) (i) Find the group of symmetries of a square. **3**

(ii) Let a permutation $\sigma \in S_n$ be written as a product of disjoint cycles. Prove that the order of σ is the l.c.m. of the lengths of its cycles. 3½

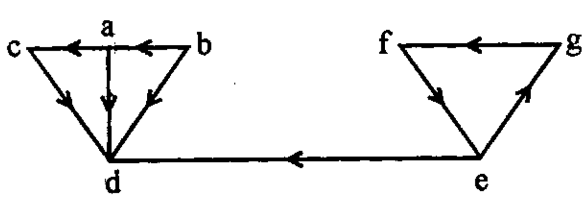
(c) (i) If A and B are sub-rings of a ring R, show that $A \cap B$ is also a sub-ring of R. 3½

(ii) Classify $f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 7 & 5 & 1 & 2 & 4 & 6 & 3 \end{pmatrix}$ as even or odd permutation. Find its inverse and order also. 3

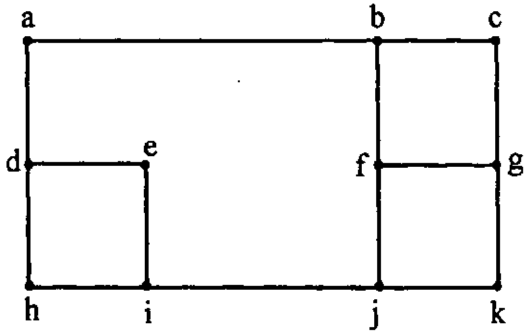
6. (a) (i) Find a maximal independent set in the following graph : 3



(ii) Find a vertex basis for the following graph 3



(b) Let the graph represents a section of a city's street map. What is the smallest number of policemen that should be positioned at corners (vertices) so that they can keep every block (edge) under surveillance ? Give a detailed logical analysis : 6



(c) Solve the traveling salesperson problem for the cost matrix : 6

	To	1	2	3	4
From	1	–	7	1	3
	2	7	–	4	5
	3	5	4	–	4
	4	1	3	6	–