This question paper contains 7 printed pages.]

Your Roll No. ....

884

B.A. Programme / II A

(T)

MATHEMATICS – Paper II

(Geometry, Differential Equations and Algebra)

(Admissions of 2004 and onwards)

Time : 3 Hours **Maximum Marks : 75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Note : The maximum marks printed on the question paper are applicable for the students of the regular colleges (Cat. 'A'). These marks will, however, be scaled up proportionately in respect of the students of NCWEB at the time of posting of awards for compilation of result.

All questions are compulsory.

Attempt any **two** parts from each question.

1. (a) Sketch the graph of the ellipse

$$16x^2 + 9y^2 - 64x - 54y + 1 = 0$$

and also label the foci, the vertices and the ends of minor axis. **6½**

(b) Consider the equation

$$9x^2 - 24xy + 16y^2 - 80x - 60y + 100 = 0$$

Rotate the coordinate axes to remove the xy-term and then identify the type of the conic represented by the equation. **6½**

(c) Identify and sketch the conic represented by the equation

$$4y^2 - x^2 + 40y - 4x + 60 = 0$$

and label the vertices, foci and asymptotes. **6½**

2. (a) (i) Find the equation of the sphere in the first octant that has radius 3 and is tangent to the three coordinate planes. **3½**

(ii) Find the equation of the sphere with diameter having end points (3, 4, 5) and (1, 2, 3). **2½**

(b) (i) Find the constant 'a' such that the vectors $\vec{u} = (2, -1, 1)$, $\vec{v} = (1, 2, -3)$, $\vec{w} = (3, a, 5)$ lie in the same plane. **3 + 3**

(ii) Using vectors, find the area of the triangle with vertices P(1, 5, –2), Q(0, 0, 0), R (3, 5, 1)

(c) Prove that the lines

$L_1 : x = 3 + 3t, y = 8 - t, z = 3 + t; -\infty < t < \infty$

$L_2 : x = -3 - 3t, y = -7 + 2t, z = 6 + 4t; -\infty < t < \infty$

are skew lines, and find the distance between them. **6**

3. (a) (i) Solve the differential equation,

$$(2xy^2 - 2y)\,dx + (3x^2y - 4x)\,dy = 0$$

by finding an integrating factor

(ii) Solve

$$x^2 \frac{d^2y}{dx^2} - 2x \frac{dy}{dx} + 2y = x^3$$ **3 + 3½**

(b) (i) Solve

$$y = 2px + p^2y$$

(ii) Find the orthogonal trajectory of the family of curves

$$Cx^2 + y^2 = 1$$ **3½ + 3**

(c) (i) Solve

$$\frac{d^2y}{dx^2} + y = \tan x$$

using the method of variation of parameters.

(ii) Verify condition of integrability for the equation

$(x - 3y - z)dx + (2y - 3x)\,dy + (z - x)\,dz = 0$

and hence solve it. **3 + 3½**

4. (a) (i) Here $p = \frac{\partial z}{\partial x}$, $q = \frac{\partial z}{\partial y}$, $r = \frac{\partial^2 z}{\partial x^2}$, $s = \frac{\partial^2 z}{\partial x\,\partial y}$ and $t = \frac{\partial^2 z}{\partial y^2}$

Form a partial differential equation by eliminating the arbitrary constants a and b from the equation

$$4z = \left(ax + \frac{y}{a} + b\right)^2$$

Examine whether the partial differential equation so obtained is linear or non-linear.

(ii) Find complete integral of the equation

$$p^2 y\,(1 + x^2) = qx^2$$

4 + 2

(b) (i) Find the general integral of the linear partial differential equation

$$(y + z)\,p + (z + x)q = x + y$$

(ii) Classify the following second order partial differential equation into elliptic, parabolic or hyperbolic

$$(x - y)\,(xr - xs - ys + yt) = (x + y)\,(p - q)$$

5 + 1

(c) Find complete integral of the equation

$$p = (z + qy)^2$$

6

5. (a) (i) Let $f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 3 & 2 & 5 & 4 & 6 & 1 & 7 \end{pmatrix}$

and $g = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 2 & 1 & 5 & 7 & 4 & 6 & 3 \end{pmatrix}$

be permutations in S_7. 3½

Compute $g^{-1}fg$. Also find orders of f and g.

(ii) Let G be a group in which $(ab)^2 = a^2b^2$ $\forall a, b \in G$. Prove that G is abelian. 3

(b) (i) State Lagrange's theorem. Hence show that if G is a finite group then $a^{o(G)} = e\ \forall a \in G$. 3½

(ii) Let S = {2, 4, 6, 8}. Show that $(S, \odot_{10})$ is an abelian group where $\odot_{10}$ is multiplication modulo 10, what is the identity element ? 3

(c) (i) Let $GL_2(R)$ be the set of all 2 × 2 invertible matrices with entries in R. Show that it forms a group under matrix multiplication. Is it an abelian group ? Justify. 3½

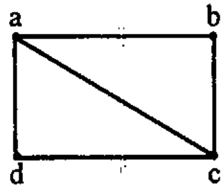
(ii) If (R, +,#) is a ring and a, b, c $\in$ R, then show that **3**

(1) $a.o = o = o.a$

(2) $a.(-b) = (-a) \cdot b = -(a.b)$

(3) $a.(b - c) = a \cdot b - a.c$

6. (a) (i) Give a model of overlapping intervals for the following graph. **2**



(ii) A is qualified to do job b. B is qualified to do either job a or b or c. C is qualified only for job b and D is qualified to do job b or d. Represent the situation by a suitable graph. Find the matching (if it exists) or explain why the matching does not exist. **4**

(b) (i) Three pitchers of sizes 8 litres, 5 litres and 3 litres are given. If initially 8 litres pitcher is full and the other two empty, find a minimal sequence of pouring so as to have exactly 4 litres of water in one pitcher. **3**

(ii) Define a latin square. Give an example of a Latin square of order 3. Is it unique ? Justify. **3**

(c) Solve the travelling salesperson problem for the cost matrix : **6**

	To	1	2	3	4
From	1	–	7	2	6
	2	1	–	8	9
	3	2	1	–	3
	4	4	2	6	–