This question paper contains 8+3 printed pages]

Your Roll No................

5271

B.A. Programme/II **B**

(I)

MATHEMATICS—Paper II

(Geometry, Differential Equations and Algebra)

(Admissions of 2004 and onwards)

Time : 3 Hours *Maximum Marks : 75*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Note :— The maximum marks printed on the question paper are applicable for the students of the regular colleges (Cat. 'A'). These marks will, however, be scaled up proportionately in respect of the students of NCWEB at the time of posting of awards for compilation of result.

Attempt *All* questions, selecting two parts from each question. Marks are as indicated.

1. (*a*) Find and sketch the equation of the hyperbola with vertices (0, ±8) and asymptotes $y = \pm \frac{4}{3}x$. Also state the reflection property of hyperbolas. 5+1

(*b*) Identify and sketch the curve $y^2 - 6y - 2x + 1 = 0$. Also label the focus, vertex and directrix. 6

(*c*) Consider the equation

$$52x^2 - 72xy + 73y^2 + 40x + 30y - 75 = 0.$$

Rotate the coordinate axes to remove the *xy*-term and then identify the type of conic represented by the equation. 6

2. (*a*) (*i*) The distance between a point P(*x*, *y*, *z*) and the point A(1, –2, 0) is twice the distance between P and the point B(0, 1, 1). Show that the set

of all such points is a sphere, and find the center and radius of the sphere.

(*ii*) Determine the surface represented by the equation $x^2 + z^2 = 1$ in 3-space. 4+2½

(*b*) Consider the points A(1, −1, 2), B(2, −3, 0), C(−1, −2, 0), D(2, 1, −1). Find the volume of the parallelopiped that has the vectors $\overrightarrow{AB}, \overrightarrow{AC}, \overrightarrow{AD}$ as adjacent edges. Also find the distance from the point D to the plane containing A, B and C. 3½+3

(*c*) Show that the lines

$L_1 : x + 1 = 4t,\ y - 3 = t,\ z - 1 = 0$

$L_2 : x + 13 = 12t,\ y - 1 = 6t,\ z - 2 = 3t$

intersect and find an equation of the plane they determine. 2½+4

3. (*a*) (*i*) Solve the differential equation

$$(5xy + 4y^2 + 1)dx + (x^2 + 2xy)dy = 0$$

by finding an integrating factor.

(*ii*) Solve the equation

$$p^2y + p(x - y) - x = 0$$

where $p = \frac{dy}{dx}$. 3½+3

(*b*) Assume that the rate at which radioactive nuclei decay is proportional to the number of such nuclei that are present in a given sample. In a certain sample, 10% of the original number of radioactive nuclei have undergone disintegration in a period of 100 years.

(*i*) What percentage of the original radioactive nuclei will remain after 1000 years ?

(*ii*) In how many years will only one fourth of the original number remain ? 4+2½

(*c*) (*i*) Find the general solution of the differential euqation

$$9x^2\frac{d^2y}{dx^2}+3x\frac{dy}{dx}+y=0.$$

(*ii*) Given that e^{-x}, e^{3x} and e^{4x} are all solutions of

$$\frac{d^3y}{dx^3}-6\frac{d^2y}{dx^2}+5\frac{dy}{dx}+12y=0.$$

Show that they are linearly independent on the interval $-\infty < x < \infty$ and write the general solution. 3½+3

4. Here $p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}, r = \frac{\partial^2 z}{\partial x^2}, s = \frac{\partial^2 z}{\partial x \partial y}, t = \frac{\partial^2 z}{\partial y^2}$

(*a*) Find the general integral of the partial differential equation

$$(2xy - 1)p + (z - 2x^2)q = 2(x - yz)$$

and also the particular integral which passes through the line $x = 1, y = 0$. 6

(*b*) (*i*) Find a complete integral of the equation

$$2(y + zq) = q(xp + yq)$$

using Charpit's method.

(*ii*) Find whether the equation

$$r + x^2 t = 0$$

is hyperbolic, parabolic or elliptic. 5+1

(c) (i) Find complete integral of the equation

$$(p + q)(z - px - qy) = 1.$$

(ii) Form the partial differential equation by eliminating the arbitrary constants a and b from the equation

$$ax^2 + by^2 + z^2 = 1.$$

Examine whether the partial differential equation so obtained is linear or non-linear. Also find its order and degree. 3+3

5. (a) (i) Let G be a group with $a, b \in G$. Assume that $o(a)$ and $o(b)$ are finite and relatively prime and that $ab = ba$. Show that :

$$o(ab) = o(a) \, . \, o(b).$$

(*ii*) Show that the set R = {2*m* : *m* is an integer} is a commutative ring for ordinary addition and multiplication. 3½+3

(*b*) (*i*) Given that f = (1325) (143) (251) is a permutation on five symbols, express it as a product of disjoint cycles. Also find the inverse of f and write it as a product of disjoint cycles.

(*ii*) Prove that the order of every element of a finite group divides the order of the group. 3+3½

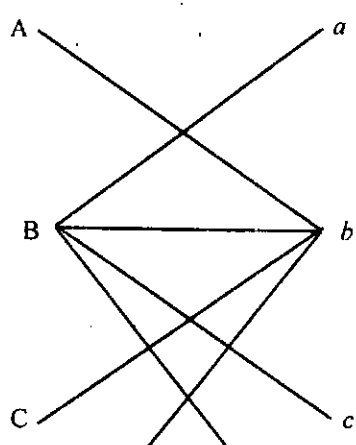
(*c*) (*i*) Show that the set Q of all rational numbers other than −1 is an abelian group w.r.t. binary composition

$$a * b = a + b + ab.$$

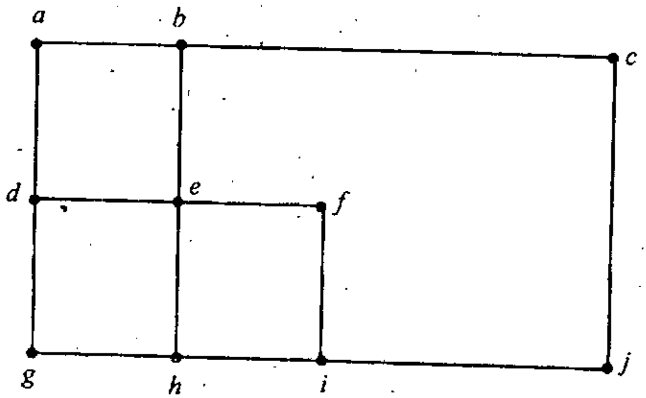
(*ii*) Prove that if G_1 and G_2 are abelian groups, then the direct product $G_1 \times G_2$ is abelian. 3+3½

6. (*a*) (*i*) Define a Latin square of order 3. Give an example of it.

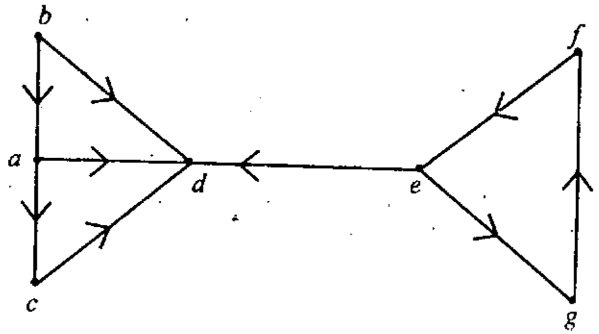
(*ii*) Find a matching or explain why none exists for the following graphs : 3+3



(*b*) (*i*) Find a maximum independent set for the following graph. What is the maximum number of independent sets needed to cover all the vertices ?



(*ii*) Find all the possible set of influential vertices for the following graphs. 3+3



(*c*) Solve the travelling salesperson problem for the given cost matrix : 6

To	1	2	3	4
From 1	—	7	2	6
2	1	—	8	9
3	2	1	—	3
4	4	2	6	—