This question paper contains 8 printed pages]

*Your Roll No.* ....................................

**5272**

**B.A. Programme/II** **B**

**(L)**

MATHEMATICS—Paper II

(Geometry, Differential Equations and Algebra)

(Admissions of 2004 and onwards)

*Time* : 3 *Hours* *Maximum Marks* : 75

*(Write your Roll No. on the top immediately on receipt of this question paper.)*

*Note* :— The maximum marks printed on the question paper are applicable for the students of the regular colleges (Cat. 'A'). These marks will, however, be scaled up proportionately in respect of the students of NCWEB at the time of posting of awards for compilation of result.

Attempt *All* questions, selecting *two* parts from each question. Marks are as indicated.

1. (*a*) Find an equation for the ellipse with length of minor axis 8 and with vertices (2, 6) and (2, –4) and also sketch it. State the reflection property of ellipses. 5+1

(*b*) Identify and sketch the curve :

$$x^2 - 4y^2 + 2x + 8y - 7 = 0.$$

Also label the vertices, foci and asymptotes. 6

(*c*) Consider the equation :

$$x^2 + 2\sqrt{3}xy + 3y^2 + 2\sqrt{3}x - 2y = 0.$$

Rotate the coordinate axes to remove the *xy*-term. Then identify the type of conic represented by the equation and sketch its graph. 6

2. (*a*) (*i*) Find an equation of the sphere with center at (2, –1, –3) and satisfying the condition that it is tangent to the *xy*-plane.

(*ii*) Determine the surface represented by the equation :

$$y^2 + z^2 = 25$$

in 3-space. 4+2½

(*b*) (*i*) Find the orthogonal projection of

$\vec{v} = 2\hat{i} - \hat{j} + 3\hat{k}$ on $\vec{b} = \hat{i} + 2\hat{j} + 2\hat{k}$ and

also find the vector component of $\vec{v}$ orthogonal

to $\vec{b}$.

(*ii*) Determine whether the vectors :

$\vec{u} = (1, -2, 1)$, $\vec{v} = (3, 0, -2)$, $\vec{w} = (5, -4, 0)$

lie in the same plane. 3½+3

(*c*) (*i*) Find the point on the line segment joining

$P_1(1, 4, -3)$ and $P_2(1, 5, -1)$ that is $\frac{2}{3}$ of the way

from $P_1$ to $P_2$.

(*ii*) Find parametric equations of the line of intersection of the planes :

$$-2x + 3y + 7z + 2 = 0$$

$$x + 2y - 3z + 5 = 0.$$ 3+3½

3. (*a*) (*i*) Solve the differential equation :

$$\left[y^2(x+1) + y\right] dx + (2xy + 1)\, dy = 0$$

by finding an integrating factor.

(*ii*) Solve :

$$y = 2px - xp^2$$

where

$$p = \frac{dy}{dx}.$$

3½+3

(*b*) (*i*) Assume that the population of a certain city increases at a rate proportional to the number of inhabitants at any time. If the population doubles in 40 years, in how many years will it triple ?

(*ii*) Show that the solutions sin 2*x*, cos 2*x* and $e^{-3x}$ of the equation :

$$\frac{d^3y}{dx^3} + 3\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 12y = 0$$

are linearly independent. 3½+3

(*c*) (*i*) Find the general solution of the differential equation :

$$x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = x^3.$$

(*ii*) Solve :

$$\frac{d^2y}{dx^2} + 4y = \sec^2 2x$$

using method of variation of parameters. 3½+3

4. Here :

$$p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}, r = \frac{\partial^2 z}{\partial x^2}, s = \frac{\partial^2 z}{\partial x \partial y}, t = \frac{\partial^2 z}{\partial y^2}.$$

(*a*) Find a complete integral of the equation :

$$p^2 x + q^2 y = z$$

using Charpit's method. 6

(*b*) (*i*) Find the general integral of the linear partial differential equation :

$$z(xp - yq) = y^2 - x^2.$$

(*ii*) Find whether the equation :

$$y^2 r - 2xys + x^2 t = \frac{y^2}{x} p + \frac{x^2}{y} q$$

is hyperbolic, parabolic or elliptic. 4+2

(*c*) (*i*) Find complete integral of the equation :

$$z = p^2 - q^2.$$

(*ii*) Eliminate the arbitrary function $f$ from the equation :

$$z = f\left(\frac{xy}{z}\right).$$

Examine whether the partial differential equation obtained by eleminating $f$ is linear or non-linear. Also find its order and degree. 3+3

5. (*a*) (*i*) Show that the set I of all integers with binary operation, defined as :

$$a \cdot b = a + b + 1 \ \forall \ a, b \in I$$

is an abelian group.

(*ii*) Let G be a group. Prove that if $x^2 = e$ for all $x \in G$, then G is abelian. 3+3½

(*b*) (*i*) Let G be a group. The set :

$$Z(G) = \{x \in G | xg = gx \ \forall \ g \in G\}$$

is called the center of G. Show that Z(G) is a subgroup of G.

(*ii*) Show that the set $\{3n : n \in Z\}$ is a commutative ring w.r.t. usual addition and multiplication.

3+3½

(*c*) (*i*) Prove that the intersection of any collection of subgroups of a group is again a subgroup.

(*ii*) Write

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 3 & 9 & 5 & 4 & 7 & 8 & 1 & 6 & 10 & 2 \end{pmatrix}$$

as

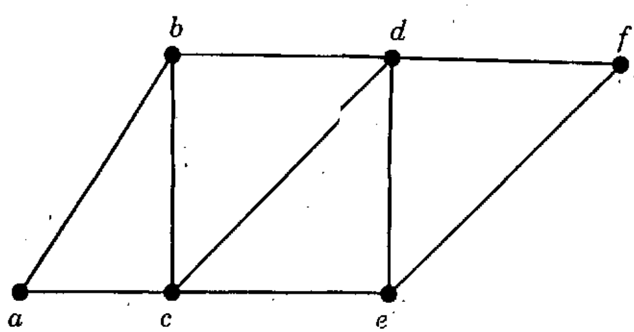
(*i*) a product of disjoint cycles

(*ii*) product of transpositions.

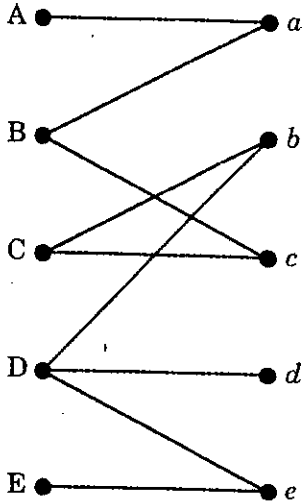
Find the inverse and order of $\sigma$. 3+3½

6. (*a*) Show that the table for any finite group (G, +) of order 4 is a Latin square of order 4 based on G. 6

(*b*) (*i*) Is the following graph an internal graph ? If yes, give a model of overlapping internals, if not, explain why not ?



(*ii*) Find a matching or explain why none exists for the following graph : 3+3




(*c*) Solve the travelling salesperson problem for the given cost matrix : 6

| From \ To | 1 | 2 | 3 | 4 | 5 |
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| **1** | — | 3 | 2 | 4 | 3 |
| **2** | 4 | — | 4 | 5 | 5 |
| **3** | 5 | 3 | — | 4 | 4 |
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| **5** | 5 | 4 | 2 | 3 | — |