This question paper contains 6 printed pages.

8224 *Your Roll No.* .....

B.A. Prog. / II **JS**

OPERATIONAL RESEARCH

Paper II— Operational Research – I

(Admissions of 2004 and onwards)

Time : 3 hours *Maximum Marks* : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

Answer six questions in all, selecting at least one question from each Section. All questions carry equal marks.

SECTION I (*Linear Programming*)

1. (*a*) Solve the following L.P.P. graphically:

Maximize $Z = x_1 + x_2$

subject to $x_1 + x_2 \leq 1$

$-3x_1 + x_2 \geq 3$

$x_1, x_2 \geq 0$

Comment on the nature of solution.

(*b*) A company has three operational departments (weaving, processing and packing) with capacity to produce three different types of clothes namely suiting, shirting and woollens yielding a profit of Rs. 2, Rs. 4 and Rs. 3 per meter respectively. One

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meter of suiting requires 3 minutes in weaving, 2 minutes in processing and 1 minute in packing. Similarly, one meter of shirting requires 4 minutes in weaving, 1 minute in processing and 3 minutes in packing. One meter of woollen requires 3 minutes in each department. In a week, total run time of each department is 60, 40 and 80 hours for weaving, processing and packing respectively. Formulate and solve the linear programming problem to find the product mix to maximize the profit.

2. (*a*) What is the significance of duality theory in linear programming? Describe the general rules for writing dual of a linear programming problem.

(*b*) Two leading firms are competing for the market share for a particular product. Each firm is considering what promotional strategy to employ for the coming period. The following pay-off matrix describes the increase in market share for firm A and decrease in market share for firm B.

Firm A's strategy	*Firm B's strategy*		
	No promotion	*Moderate promotion*	*Heavy promotion*
No promotion	0	2	−15
Moderate promotion	12	8	−4
Price cut	20	15	6

Find the optimum strategies and the worth of such strategies for the two firms.

3. (*a*) There are four jobs, A, B, C, and D and these are to be performed on four machines, I, II, III and IV. The processing costs are given in the following table:

	Machines			
Jobs	*I*	*II*	*III*	*IV*
A	200	150	170	220
B	160	120	150	140
C	190	195	190	200
D	180	175	160	190

Suggest optimal assignment of jobs to the machines so that total cost of processing is minimum.

(*b*) A company has factories at F_1, F_2, and F_3, which supply warehouses at W_1, W_2 and W_3. Weekly factory capacities are 14, 12, and 5 (in '000) units respectively. Weekly warehouse requirements are 6, 10 and 15 (in '000) units respectively. Unit shipping costs (in rupees) are as follows:

From \ *To*	W_1	W_2	W_3	*Supply*
F_1	6	8	4	14
F_2	4	9	8	12
F_3	1	2	6	5
	6	10	15	31

Determine the optimum distribution plan for this company to minimize shipping cost.

SECTION II (*Inventory Management*)

4. (*a*) What do you understand by an inventory system? Classify and discuss the different types of costs involved in inventory system.

   (*b*) A shopkeeper has a uniform demand of an item at the rate of 600 items per year. He buys from a supplier at a cost of Rs. 8 per item and the cost of ordering is Rs. 12 each time. If the stockholding costs are 20% per year of stock value, how frequently should he replenish his stocks and what is the optimal order quantity?

5. (*a*) Obtain an expression for the EOQ for an inventory system in which supply is instantaneous, demand rate is uniform and backorders are allowed and are fully backlogged.

   (*b*) An item is produced at the rate of 50 items per day. The demand occurs at the rate of 25 items per day. If the set-up cost is Rs. 100 and holding cost is Rs. 0·01 per unit of item per day, find the economic production quantity, assuming that shortages are not permitted. Also find cycle time and minimum total cost.

6. (*a*) Find the optimum order quantity for a probabilistic discrete inventory model with

uniform demand. Shortages are allowed and backlogged fully. Set-up cost per period is negligible.

(*b*) Write a note on ABC analysis.

SECTION III (*Queueing Theory*)

7. (*a*) Discuss various characteristics of the queueing system.

(*b*) Show that inter-arrival times are distributed exponentially if arrivals follow Poisson process.

8. For a (M|M|C) : (∞|FIFO) queueing model, derive the steady state expressions for:

(*i*) Probability that a customer will not have to wait

(*ii*) Expected number of customers in the queue

(*iii*) Expected waiting time of a customer in the system.

9. (*a*) The rate of arrival of customers at a public telephone booth follows Poisson distribution with an average time of 10 minutes between one customer and the next. The duration of a phone call is assumed to follow exponential distribution, with mean time of 3 minutes.

(*i*) What is the probability that a person arriving at the booth will have to wait?

(*ii*) What is the average length of non-empty queue that forms from time to time?

(*b*) A supermarket has two persons serving at the counters. The customers arrive in a Poisson fashion at the rate of 12 per hour. The service time for each customer follows exponential distribution, with mean time of 6 minutes. Find:

(*i*) The probability that an arriving customer has to wait for the service.

(*ii*) The average number of customers in the system.

(*iii*) The average time spent by a customer in the supermarket.