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Your Roll No. .....

1182

B.A. (Programme)/II **C**

MATHEMATICS– Paper II

(Geometry, Differential Equations and Algebra)

(Admissions of 2004 and onwards)

Time : 3 *Hours* *Maximum Marks* : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

Note :— The maximum marks printed on the question paper are applicable for the students of the regular colleges (Cat. 'A'). These marks will, however, be scaled up proportionately in respect of the students of NCWEB at the time of posting of awards for compilation of result.

All questions are compulsory.

Attempt any *two* parts from each question.

P.T.O.

1. (*a*) Describe and sketch the conic

$$x^2 \quad y^2 - 4x - 8y \quad 21 = 0$$

Find the centre, the vertices and the foci. 6

(*b*) Find an equation of the parabola with vertex at (1, 1) and directrix $y \quad -2$.

Also sketch it. 6

(*c*) Consider the equation

$$9x^2 \quad 24xy + 16y^2 \quad 80x \quad 60y + 100 = 0$$

Rotate the coordinate axes to remove the xy-term and then identify the type of the conic represented by the equation. 6

2. (*a*) Show that the plane

$$2x - 2y + z + 12 = 0$$

touches the sphere

$$x^2 + y^2 + z^2 - 2x - 4y + 2z \quad 3$$

and find the point of contact. 6½

(*b*) (*i*) Determine whether the line 2½

$$x = 2t,\ y = 2t,\ z = 3t$$

is parallel to the plane

$$x - y + 2z = 5.$$

(*ii*) Using vectors find the area of the triangle with vertices : 4

P(1, 5, –2), Q(0, 0, 0) and R(3, 5, 1).

(*c*) Show that the lines : 6½

$L_1 : x = 2 + 8t,\ y = 6 - 8t,\ z = 10t \qquad -\infty < t < \infty$

$L_2 : x = 3 + 8t,\ y = 5 - 3t,\ z = 6 + t \qquad -\infty < t < \infty$

are skew lines and find the distance between them.

3. (*a*) Here $D = \dfrac{d}{dx}$

(*i*) Using the method of variation of parameters, solve : 4

$$\left(D^2 - 1\right) y = \frac{2}{1 + e^x}.$$

(*ii*) Solve : 3½

$$(x^4 - y^4)dx - xy^3\, dy = 0.$$

P.T.O.

(*b*) (*i*) Find the orthogonal trajectories of the family curves $xy = c$, where c is an arbitrary constant. 4

(*ii*) Solve : 3½

$$(2xz - yz)\,dx + (2yz - zx)\,dy - (x^2 - xy + y^2)dz$$

(*c*) (*i*) Solve : 3

$$\frac{dx}{y^3x - 2x^4} = -\frac{dy}{2y^4 - x^3y} = \frac{dz}{9z\left(x^3 - y^3\right)}.$$

(*ii*) Show that linearly independent solutions of

$$y'' - 2y' + 2y = 0$$

are $e^x \sin x$ and $e^x \cos x$.

What is the general solution ? Find the solution $y(x)$ with the property : 4½

$$y(0) = 2,\ y'(0) = -3.$$

4. (*a*) Solve the partial differential equation 5

$$(y + zx)\, p - (x + yz)\, q = x^2 - y^2$$

where

$$p = \frac{\partial z}{\partial x},\ q = \frac{\partial z}{\partial y}.$$

(*b*) Find the complete integral of the equation 5

$$p^2 q(x^2 + y^2) = p^2 + q.$$

(*c*) Formulate a partial differential equation by eliminating the arbitrary constants b and c from the equation

$$x^2 + (y - b)^2 + (z - c)^2 = 1.$$ 5

5. (*a*) Consider the following permutation in S_8 :

$$f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 6 & 5 & 2 & 4 & 3 & 1 & 7 \end{pmatrix}$$

$$g = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 3 & 1 & 4 & 7 & 2 & 5 & 8 & 6 \end{pmatrix}$$

Compute $g^{-1} fg$. Also find the order of f and g. 6½

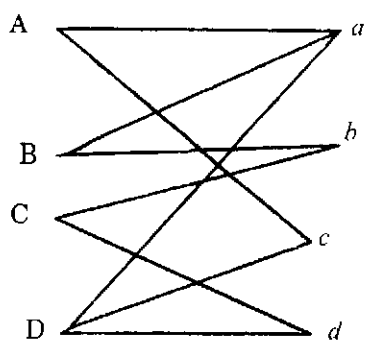
(*b*) Prove that the set G = {1, 2, 3, 4, 5, 6} is a finite abelian group of order 6 with respect to multiplication module 7. 6½

(*c*) (*i*) If *a* and *b* are any two elements of a group G, then show that $(bab^{-1})^n = ba^nb^{-1}$ for any integer *n*. 3

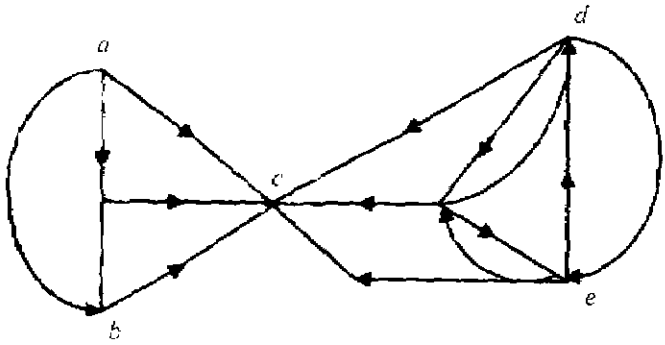
(*ii*) Define a ring and give an example of a non-commutative ring with unity. 3½

6. (*a*) Define orthogonal Latin squares. Construct tree mutually orthogonal Latin square of order four. 6

(*b*) (*i*) Find the matching for the following graph : 3



(*ii*) Find the minimal set of vertices for the following directed graph : 3



(*c*) Solve the following travelling salesperson problem : 6

From \ To	1	2	3	4	5	6
1	–	3	3	2	7	3
2	3	–	3	4	5	5
3	3	3	–	1	4	4
4	2	4	1	–	5	5
5	7	5	4	5	–	4
6	3	5	4	5	4	–